Fixed Income

Topic Weight on Exam	10–20%
SchweserNotes^{TM} Reference	Book 4, Pages 1–105

The Term Structure and Interest Rate Dynamics
Cross-Reference to CFA Institute Assigned Topic Review #35

This topic review discusses the theories and implications of the term structure of interest rates. In addition to understanding the relationships between spot rates, forward rates, yield to maturity, and the shape of the yield curve, be sure you become familiar with concepts like the *z*-spread, the TED spread, and the LIBOR-OIS spread and key rate duration.

Spot Rates and Forward Rates

The spot rate for a particular maturity is equal to a geometric average of the current one-period spot rate and a series of one-period forward rates.

$$[1 + S_{(j + k)}]^{(j + k)} = (1 + S_j)^j[1 + f(j,k)]^k$$

When the spot curve is flat, forward rates will equal spot rates. When the spot curve is upward sloping (downward sloping), forward rate curves will be above (below) the spot curve, and the yield for a maturity of *T* will be less than (greater than) the spot rate S_T.

Evolution of Spot Rates in Relation to Forward Rates

If spot rates evolve as predicted by forward rates, bonds of all maturities will realize a one-period return equal to the one-period spot rate, and the forward price will remain unchanged.

Active bond portfolio management is built on the presumption that the current forward curve may not accurately predict future spot rates. Managers attempt to outperform the market by making predictions about how spot rates will evolve relative to the rates suggested by forward rate curves.

If an investor believes that future spot rates will be lower than corresponding forward rates, the investor will purchase bonds (at a presumably attractive price) because the market appears to be discounting future cash flows at "too high" a discount rate.

"Riding the Yield Curve"

When the yield curve is upward sloping, bond managers may use the strategy of "riding the yield curve" to chase above-market returns. By holding long-maturity (relative to their investment horizon) bonds, the manager earns an excess return as the bond "rolls down the yield curve" (i.e., approaches maturity and increases in price). As long as the yield curve remains upward sloping, this strategy will add to the return of a bond portfolio.

The Swap Rate Curve

The swap rate curve provides a benchmark measure of interest rates. It is similar to the yield curve except that the rates used represent the interest rates of the fixed-rate leg in an interest rate swap.

Market participants prefer the swap rate curve as a benchmark interest rate curve rather than a government bond yield curve for the following reasons:

- Swap rates reflect the credit risk of commercial banks rather than governments.
- The swap market is not regulated by any government.
- The swap curve typically has yield quotes at many maturities.

Institutions like wholesale banks are familiar with swaps and, as a result, often use swap curves (rather than other interest rate benchmarks) to value their assets and liabilities.

We define swap spread as the additional interest rate paid by the fixed-rate payer of an interest rate swap over the rate of the "on-the-run" government bond of the same maturity.

swap spread = (swap rate) − (Treasury bond yield)

The *Z*-spread

The *Z*-spread is the spread that, when added to each spot rate on the yield curve, makes the present value of a bond's cash flows equal to the bond's market price. The *Z* refers to zero volatility—a reference to the fact that the *Z*-spread assumes

interest rate volatility is zero. *Z*-spread is not appropriate to use to value bonds with embedded options.

The TED spread

TED spread = (three-month LIBOR rate) – (three-month T-bill rate)

The TED spread is used as an indication of the overall level of credit risk in the economy.

The LIBOR-OIS Spread

The LIBOR-OIS spread is the amount by which the LIBOR rate (which includes credit risk) exceeds the overnight indexed swap (OIS) rate (which includes only minimal credit risk). The LIBOR-OIS spread is a useful measure of credit risk and provides an indication of the overall well-being of the banking system.

Traditional Theories of the Term Structure of Interest Rates

There are several traditional theories that attempt to explain the term structure of interest rates:

Unbiased expectations theory—Forward rates are an unbiased predictor of future spot rates. Also known as the pure expectations theory.

Local expectations theory—Preserves the risk-neutrality assumption only for short holding periods, while over longer periods, risk premiums should exist. This implies that over short time periods, every bond (even long-maturity risky bonds) should earn the risk-free rate.

Liquidity preference theory—Investors demand a liquidity premium that is positively related to a bond's maturity.

Segmented markets theory—The shape of the yield curve is the result of the interactions of supply and demand for funds in different market (i.e., maturity) segments.

Preferred habitat theory—Similar to the segmented markets theory, but recognizes that market participants will deviate from their preferred maturity habitat if compensated adequately.

Modern Term Structure Models

Two major classes of these modern term structure models are:

1. **Equilibrium term structure models**
    *   **Cox-Ingersoll-Ross (CIR) model:**

        $$dr = a(b - r)dt + \sigma\sqrt{r}dz$$

        Assumes the economy has a natural long-run interest rate (b) that the short-term rate (r) converges to.

    *   **Vasicek model:**

        $$dr = a(b - r)dt + \sigma dz$$

        Similar to the CIR model but assumes that interest rate volatility level is independent of the level of short-term interest rates.

2. **Arbitrage-free models**—Begins with observed market prices and the assumption that securities are correctly priced.
    *   **Ho-Lee model:**

        $$dr_t = \theta_t dt + \sigma dz_t$$

        This model is calibrated by using market prices to find the time-dependant drift term θ_t that generates the *current* term structure.

Managing Bond Exposure to the Factors Driving the Yield Curve

We can measure a bond's exposure to the factors driving the yield curve in a number of ways:

1. **Effective duration**—Measures the sensitivity of a bond's price to parallel shifts in the benchmark yield curve.

2. **Key rate duration**—Measures bond price sensitivity to a change in a specific par rate, keeping everything else constant.

3. **Sensitivity to parallel, steepness, and curvature movements**—Measures sensitivity to three distinct categories of changes in the shape of the benchmark yield curve.

The Arbitrage-Free Valuation Framework
Cross-Reference to CFA Institute Assigned Topic Review #36

This topic review introduces valuation of fixed-income securities using spot rates, as well as using the backward induction methodology in a binomial interest rate tree framework.

Valuation of bonds using a zero-coupon yield curve (also known as the spot rate curve) is suitable for option-free bonds. However, for bonds with embedded options where the value of the option varies with outcome of unknown forward rates, a model that allows for variability of forward rates is necessary. One such model is the binomial interest rate tree framework.

Binomial Interest Rate Tree Framework

The binomial interest rate tree framework is a lognormal random walk model with two equally likely outcomes for one-period forward rates at each node. A volatility assumption drives the spread of the nodes in the tree. The tree is calibrated such that (1) the values of benchmark bonds using the tree are equal to the bonds' market prices, (2) adjacent forward rates at any nodal period are two standard deviations apart, and (3) the midpoint for each nodal period is approximately equal to the implied one-period forward rate for that period.

Backward induction is the process of valuing a bond using a binomial interest rate tree. The term *backward* is used because in order to determine the value of a bond at Node 0, we need to know the values that the bond can take on at nodal period 1, and so on.

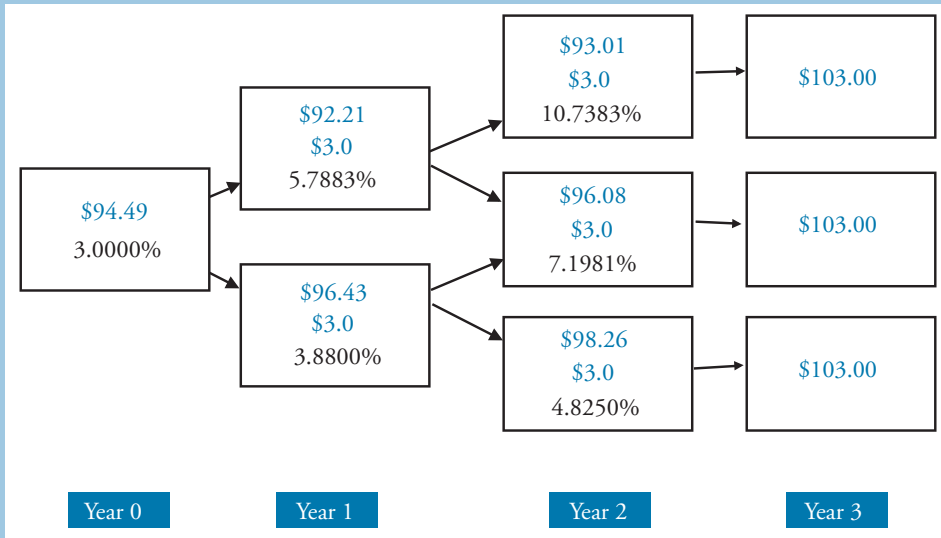> **Example: Valuation of option-free bond using binomial tree**
>
> Samuel Favre is interested in valuing a three-year, 3% annual-pay Treasury bond. Favre wants to use a binomial interest rate tree with the following rates:

One-Period Forward Rate in Year

0	1	2
3%	5.7883%	10.7383%
	3.8800%	7.1981%
		4.8250%

Compute the value of the $100 par option-free bond.

Answer:

$$V_{2,UU} = \frac{103}{(1.107383)} = \$93.01$$

$$V_{2,UL} = \frac{103}{(1.071981)} = \$96.08$$

$$V_{2,LL} = \frac{103}{(1.048250)} = \$98.26$$

$$V_{1,U} = \frac{1}{2} \times \left[\frac{93.01+3}{1.057883} + \frac{96.08+3}{1.057883} \right] = \$92.21$$

$$V_{1,L} = \frac{1}{2} \times \left[\frac{96.08+3}{1.038800} + \frac{98.26+3}{1.038800} \right] = \$96.43$$

$$V_0 = \frac{1}{2} \times \left[\frac{92.21+3}{1.03} + \frac{96.43+3}{1.03} \right] = \$94.485$$

Pathwise Valuation in a Binomial Interest Rate Framework

In the pathwise valuation approach, the value of the bond is simply the average of the values of the bond at each path. For an n-period binomial tree, there are $2^{(n-1)}$ possible paths.

Monte Carlo Forward-Rate Simulation

The Monte Carlo simulation method uses pathwise valuation and a large number of randomly generated simulated paths. Mortgage-backed securities (MBS) have path-dependent cash flows due to their embedded prepayment option. The Monte Carlo simulation method should be used for valuing MBS as the binomial tree backwards-induction process is inappropriate for securities with path-dependent cash flows.

Valuation and Analysis: Bonds with Embedded Options
Cross-Reference to CFA Institute Assigned Topic Review #37

This topic review extends the arbitrage-free valuation framework to valuation of bonds with embedded options. Make sure you understand the risk/return dynamics of embedded options, including their impact on a bond's duration and convexity. Master the concept of OAS and *Z*-spread. Finally, understand the risk/return characteristics of convertibles.

Callable and Putable Bonds, Straight Bonds, and Embedded Options

Value of an option embedded in a callable or putable bond:

$$V_{call} = V_{straight} - V_{callable}$$

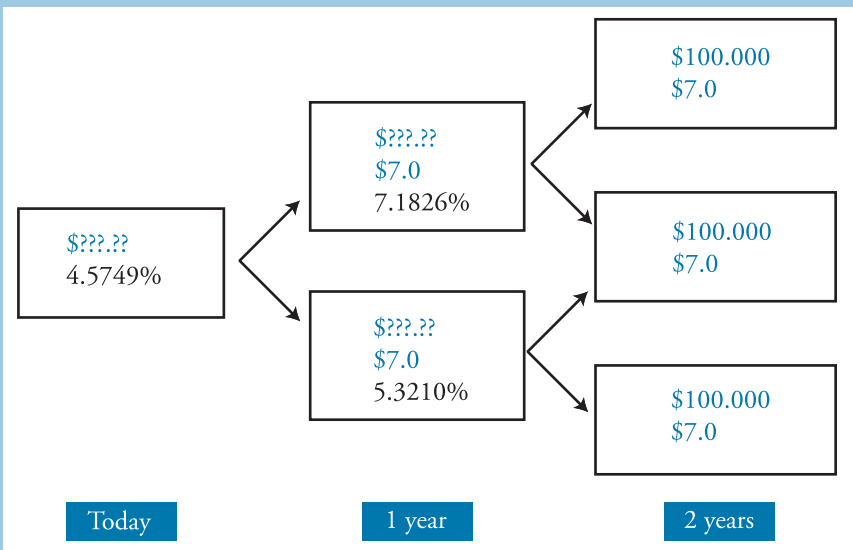$$V_{put} = V_{putable} - V_{straight}$$

Valuing a Bond with Embedded Options Using Backward Induction

Example: Valuation of call and put options

Consider a two-year, 7% annual-pay, $100 par bond callable in one year at $100. Also consider a two-year, 7% annual-pay, $100 par bond putable in one year at $100.

The interest rate tree at 15% assumed volatility is as given below.

Value the embedded call and put options.

Answer:

Value of the straight (option-free) bond:

Consider the value of the bond at the *upper* node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \times \left[\frac{\$100 + \$7}{1.071826} + \frac{\$100 + \$7}{1.071826} \right] = \$99.830$$

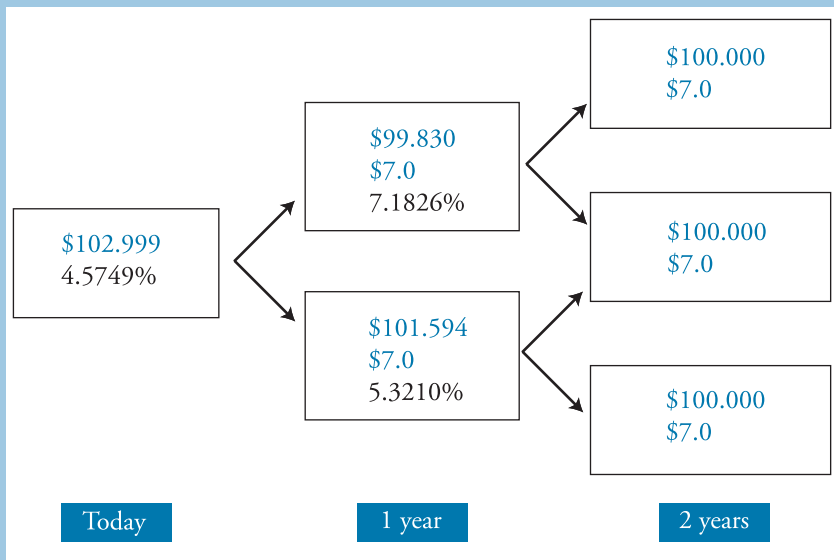Similarly, the value of the bond at the *lower* node for Period 1, $V_{1,L}$, is:

$$V_{1,L} = \frac{1}{2} \times \left[\frac{\$100 + \$7}{1.053210} + \frac{\$100 + \$7}{1.053210} \right] = \$101.594$$

Now calculate V_0, the current value of the bond at Node 0.

$$V_0 = \frac{1}{2} \times \left[\frac{\$99.830 + \$7}{1.045749} + \frac{\$101.594 + \$7}{1.045749} \right] = \$102.999$$

The completed binomial tree is shown below:

Valuing a Two-Year, 7.0% Coupon, Option-Free Bond

Value of the callable bond:

The call rule (call the bond if the price exceeds $100) is reflected in the boxes in the completed binomial tree, where the second line of the boxes at the one-year node is the lower of the call price or the computed value. For example, the value of the bond in one year at the lower node is $101.594. However, in this case, the bond will be called, and the investor will only receive $100. Therefore, for valuation purposes, the value of the bond in one year at this node is $100.
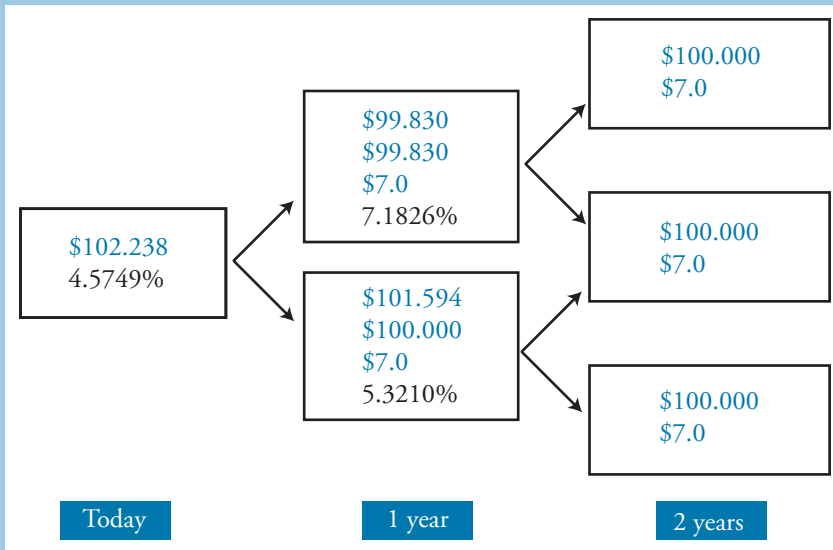
$$V_{1,L} = \$100$$

$$V_{1,U} = (107 / 1.071826) = \$99.830$$

The calculation for the current value of the bond at Node 0 (today), assuming the simplified call rules of this example, is:

$$V_0 = \frac{1}{2} \times \left[\frac{\$99.830 + \$7}{1.045749} + \frac{\$100.00 + \$7}{1.045749} \right] = \$102.238$$

The completed binomial tree is shown below:

Valuing a Two-Year, 7.0% Coupon, Callable Bond, Callable in One Year at 100

Value of the putable bond:

Similarly, for a putable bond, the put rule is to put the bond if the value falls below $100. The put option would therefore be exercised at the upper-node in year 1 and hence the $99.830 computed value is replaced by the exercise price of $100.

$V_{1,U} = 100$

$V_{1,L} = (107 / 1.053210) = \$101.594$

$$V_0 = \frac{1}{2} \times \left[\frac{100 + 7}{1.045749} + \frac{101.594 + 7}{1.045749} \right] = \$103.081$$

Value of the embedded options:

$V_{call} = V_{straight} - V_{callable} = \$102.999 - \$102.238 = \$0.76$

$V_{put} = V_{putable} - V_{straight} = \$103.081 - \$102.999 = 0.082$

Impact on Values

When interest rate volatility increases, the value of both call and put options on bonds increase. As volatility increases, the value of a callable bond decreases (remember that the investor is *short* the call option) and the value of a putable bond increases (remember that the investor is *long* the put option).

The short call in a callable bond limits the investor's upside when rates decrease, while the long put in a putable bond hedges the investor against rate increases.

The value of the call option will be lower in an environment with an upward-sloping yield curve because the probability of the option going in the money is low. A call option gains value when the upward-sloping yield curve flattens. Conversely, a put option will have a higher probability of going in the money when the yield curve is upward sloping; the option loses value if the upward-sloping yield curve flattens.

Option-Adjusted Spreads

The option-adjusted spread (OAS) is the constant spread added to each forward rate in a benchmark binomial interest rate tree, such that the sum of the present values of a credit risky bond's cash flows equals its market price. (The actual computation of OAS is an iterative process outside the scope of the curriculum.)

Binomial trees generated under an assumption of high interest rate volatility will lead to higher values for a call option and a corresponding lower value for a callable bond. Under a high volatility assumption, we would already have a lower computed value for the callable bond and, hence, the additional spread (i.e., the OAS) needed to force the discounted value to equal the market price will be lower.

When an analyst uses a lower-than-actual (higher-than-actual) level of volatility, the *computed* OAS for a *callable* bond will be too high (too low) and the callable bond will be erroneously classified as underpriced (overpriced).

Similarly, when the analyst uses a lower-than-actual (higher-than-actual) level of volatility, the *computed* OAS for a *putable* bond will be too low (high) and the putable bond will be erroneously classified as overpriced (underpriced).

Effective Duration

$$\text{effective duration} = ED = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

effective duration (callable/putable) $\leq$ effective duration (straight)

effective duration (zero) $\approx$ maturity of the bond

effective duration of floater $\approx$ time in years to next reset

Evaluating the Interest Rate Sensitivity of Bonds with Embedded Options

For bonds with embedded options, one-sided durations—durations when interest rates rise versus when they fall—are better at capturing interest rate sensitivity than regular effective durations. When the underlying option is at- or near-the-money, callable (putable) bonds will have lower (higher) one-sided down-duration than one-sided up-duration.

Callable bonds with low coupon rates are unlikely to be called and, hence, their maturity-matched rate is their most critical rate (and will be the highest key rate duration).

As the coupon rate increases, a callable bond is more likely to be called, and the time-to-exercise rate will start dominating the time-to-maturity rate.

Putable bonds with high coupon rates are unlikely to be put and are most sensitive to its maturity-matched rate. As the coupon rate decreases, a putable bond is more likely to be put and the time-to-exercise rate will start dominating the time-to-maturity rate.

Effective Convexities of Callable, Putable, and Straight Bonds

Straight and putable bonds exhibit positive convexity throughout. Callable bonds also exhibit positive convexity when rates are high. However, at lower rates, callable bonds exhibit negative convexity.

Defining Features of a Convertible Bond

The owner of a convertible bond can exchange the bond for the common shares of the issuer; it includes an embedded call option giving the bondholder the right to buy the common stock of the issuer.

Components of a Convertible Bond's Value

The conversion ratio is the number of common shares for which a convertible bond can be exchanged.

conversion value = market price of stock × conversion ratio

market conversion price = market price of convertible bond / conversion ratio

market conversion premium per share = market conversion price – market price

The minimum value at which a convertible bond trades is its straight value or its conversion value, whichever is greater.

Valuing a Convertible Bond in an Arbitrage-Free Framework

The value of a bond with embedded options can be calculated as the value of the straight bond plus (minus) the value of options that the investor is long (short).

callable and putable convertible bond value = straight value of bond
+ value of call option on stock
– value of call option on bond
+ value of put option on bond

Risk–Return Characteristics of a Convertible Bond

- The major benefit from investing in convertible bonds is the price appreciation resulting from an increase in the value of the common stock.
- The main drawback of investing in a convertible bond versus investing directly in the stock is that when the stock price rises, the bond will underperform the stock because of the conversion premium of the bond.
- If the stock price remains stable, the return on the bond may exceed the stock returns due to the coupon payments received from the bond.
- If the stock price falls, the straight value of the bond limits downside risk (assuming bond yields remain stable).

Credit Analysis Models
Cross-Reference to CFA Institute Assigned Topic Review #38

Credit Risk Measures

Probability of default is the probability that a borrower (i.e., the bond issuer) fails to pay interest or repay principal when due. **Loss given default** refers to the value a bond investor will lose if the issuer defaults. **Expected loss** is equal to the probability of default multiplied by the loss given default.

The **present value of expected loss** adjusts the expected loss measure by incorporating time value and by using risk-neutral probabilities instead of default probabilities. The present value of expected loss is the difference between the value of a credit-risky bond and an otherwise identical risk-free bond.

Credit Ratings and Credit Scores

Credit ratings and credit scores are ordinal rankings of credit quality. While simple and easy to understand, credit ratings do not adjust with business cycles, and the stability in ratings comes at the expense of a reduction in correlation with default probabilities.

Structural Models of Corporate Credit Risk

Structural models of corporate credit risk are based on the structure of a company's balance sheet and rely on insights provided by option pricing theory. Stock of a company with risky debt outstanding can be viewed as a call option on the company's assets. If the value of the assets exceeds the face value of the debt, the shareholders receive the residual after paying the debt investors. If, on the other hand, the assets are insufficient to cover the face value of the debt, the value of the stock is zero (due to limited liability).

$$\text{value of stock}_T = \max[A_T - K, 0]$$

$$\text{value of debt}_T = \min[K, A_T]$$

where:
A_T = value of company's assets at maturity of debt (at t = T)
K = face value of debt

Debt investors can also be thought of as being short a put option on company assets; when the assets are insufficient to cover the face value of debt, shareholders can exercise the put option to sell the assets at face value to pay off the debt.

value of risky debt
= value of risk-free debt – value of a put option on the company's assets

Structural models assume that the company's assets trade in a frictionless market with return of μ and variance of σ^2 (this assumption severely limits the utility of structural models). The risk-free rate is also assumed to be constant and the company's balance sheet is assumed to be simple (i.e., only a single issue of risky debt).

Because historical asset returns are not available, implicit estimation techniques are needed for input parameters of the structural models.

Reduced Form Models of Corporate Credit Risk

Reduced form models do not impose assumptions on the company's balance sheet; instead, they impose assumptions on the output of a structural model. Reduced form models also allow the analyst flexibility to incorporate real world conditions in the model. The inputs for reduced form models can be estimated using historical data; this is called hazard rate estimation. Reduced form models assume that the company has at least one issue of risky zero-coupon debt outstanding. The risk-free rate, probability of default, and recovery rate are all allowed to vary with the state of the economy.

A major strength (and advantage over structural models) is that input estimates are observable and hence historical estimation procedures can be utilized. However, the model should be back-tested properly, otherwise the hazard rate estimation procedures (using past observations to predict the future) may not be valid.

Term Structure of Credit Spreads

Credit spread is the difference between the yield on a zero-coupon *credit-risky* bond and the yield on a zero-coupon *risk-free* bond. The term structure of credit spread captures the relationship between credit spread and maturity.

Present Value of Expected Loss on a Bond

Present value of expected loss is the difference between the value of a risk-free bond and the value of a similar risky bond. This is the maximum amount an investor would pay an insurer to bear the credit risk of a risky bond. We can estimate the present value of expected loss from the credit spread on a risky bond (given the risk-free rate).

Credit Analysis of ABS vs. Credit Analysis of Corporate Debt

Unlike corporate debt, ABS do not default; rather, losses in an ABS's collateral pool are borne by different tranches of the ABS structure based on the distribution waterfall. Hence, credit analysis of ABS entails evaluation of the collateral pool as well as the distribution waterfall. For this reason, the concept of probability of default does not apply to ABS; instead, we use the probability of loss.

Credit Default Swaps
Cross-Reference to CFA Institute Assigned Topic Review #39

Credit Default Swaps

A **credit default swap** (CDS) is a contract between two parties in which one party purchases protection from the other party against losses from the default of a borrower. If a credit event occurs, the *credit protection buyer* gets compensated by the *credit protection seller*. To obtain this coverage, the protection buyer pays the seller a premium called the *CDS spread*. The protection seller is assuming (i.e., long) credit risk, while the protection buyer is short credit risk.

The payoff on a single-name CDS is based on the market value of the *cheapest-to-deliver* (CTD) bond that has the same seniority as the reference obligation. Upon default, a single-name CDS is terminated.

An **index CDS** covers an equally weighted combination of borrowers. When one of the index constituents defaults, there is a payoff and the notional principal is adjusted downward.

CDS Pricing

The factors that influence the pricing (i.e., spread) of CDS include the probability of default, the loss given default, and the coupon rate on the swap. The conditional probability of default (i.e., the probability of default given that default has not already occurred) is called the hazard rate.

$$\text{expected loss}_t = (\text{hazard rate})_t \times (\text{loss given default})_t$$

If the coupon payment on the swap is not set to be equal to the credit spread of the reference obligation, an upfront payment from one of the counterparties to the other is necessary.

$$\text{upfront payment (by protection buyer)} = PV(\text{protection leg}) - PV(\text{premium leg})$$

or

$$\text{upfront premium} \approx (\text{CDS spread} - \text{CDS coupon}) \times \text{CDS duration}$$

After inception of the swap, the value of the CDS changes as the spread changes.

profit for protection buyer (%) ≈ change in spread (%) × CDS duration

CDS Uses

In a naked CDS, an investor with no exposure to the underlying purchases protection in the CDS market. In a long/short trade, an investor purchases protection on one reference entity while selling protection on another reference entity.

A curve trade is a type of long/short trade where the investor is buying and selling protection on the same reference entity but with different maturities. An investor who believes the short-term outlook for the reference entity is better than the long-term outlook can use a curve-steepening trade (buying protection in a long-term CDS and selling protection in a short-term CDS) to profit if the credit curve steepens. Conversely, an investor who is bearish about the reference entity's prospects in the short term will enter into a curve-flattening trade.