(d) The nominal moment of resistance is

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = \frac{(0.22\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)(5.78\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}}$$
$$= 6.36\ \text{ft-kips}$$

(e) The limiting reinforcement ratio for a tension-controlled section is

$$\rho_t = 0.319\beta_1\frac{f_c'}{f_y} = (0.319)(0.85)\left(\frac{3000\ \frac{\text{lbf}}{\text{in}^2}}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right)$$
$$= 0.0136$$

The reinforcement ratio provided is

$$\begin{aligned}\rho &= \frac{A_s}{bd} \\ &= \frac{0.22\ \text{in}^2}{(12\ \text{in})(6\ \text{in})} \\ &= 0.003 \\ &< \rho_t\end{aligned}$$

The section is tension-controlled and the strength reduction factor is

$$\phi = 0.9$$

The maximum permissible factored moment is

$$\begin{aligned}M_u &= \phi M_n = (0.9)(6.36\ \text{ft-kips}) \\ &= 5.73\ \text{ft-kips}\end{aligned}$$

(f) The applied factored dead load moment is

$$\begin{aligned}M_{uD} &= \frac{1.2 w_D l^2}{8} \\ &= \frac{(1.2)\left(0.12\ \frac{\text{kip}}{\text{ft}}\right)(12\ \text{ft})^2}{8} \\ &= 2.59\ \text{ft-kips}\end{aligned}$$

(g) From ACI Eq. 5.3.1b, the maximum permissible strength level live load moment is

$$\begin{aligned}M_{uL} &= M_u - M_{uD} \\ &= 5.73\ \text{ft-kips} - 2.59\ \text{ft-kips} \\ &= 3.14\ \text{ft-kips}\end{aligned}$$

(h) The maximum permissible service level live load moment is

$$\begin{aligned}M_L &= \frac{M_{uL}}{1.6} = \frac{3.14\ \text{ft-kips}}{1.6} \\ &= 1.96\ \text{ft-kips}\end{aligned}$$

(i) The permissible service level live load is

$$\begin{aligned}w_L &= \frac{8M_L}{l^2} \\ &= \frac{(8)(1.96\ \text{ft-kips})\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)}{(12\ \text{ft})^2} \\ &= 109\ \text{lbf/ft}\end{aligned}$$

Design Procedure for a Singly Reinforced Beam

The procedure to select a suitable section to resist a given bending moment, M_u, consists of the following steps.

step 1: Assume beam dimensions and concrete strength.

step 2: Calculate the design moment factor from

$$K_u = \frac{M_u}{bd^2}$$

step 3: Calculate the ratio

$$\frac{K_u}{f_c'}$$

step 4: Assume a tension-controlled section, since generally this is the case, and determine the reinforcement index, ω, from App. 2.A.

step 5: Determine the required reinforcement from

$$\rho = \frac{\omega f_c'}{f_y}$$

step 6: Check that the beam complies with the maximum reinforcement requirements of ~~ACI Sec. 9.3.3.1.~~

$$\rho \le 0.364\beta_1\frac{f_c'}{f_y}$$

Increase the beam size or f_c' if necessary.

step 7: Check that the beam complies with tension-controlled reinforcement requirements of ~~ACI Sec. 9.3.3.1.~~

$$\rho \leq 0.319\beta_1 \frac{f_c'}{f_y}$$

Increase the beam size or f_c' if necessary.

step 8: Check that the beam complies with minimum reinforcement requirements of ACI Sec. 9.6.1.2.

$$\rho_{\text{min}} = \frac{3\sqrt{f_c'}}{f_y} > \frac{200}{f_y}$$

Increase the beam size or f_c' if necessary.

Example 2.3

A reinforced concrete beam with an effective depth of 16 in and a width of 12 in is reinforced with grade 60 bars and has a concrete compressive strength of 3000 lbf/in^2. Determine the area of tension reinforcement required if the beam supports a total factored moment of 150 ft-kips.

Solution

The design moment factor is

$$K_u = \frac{M_u}{bd^2} = \frac{(150 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(12 \text{ in})(16 \text{ in})^2}$$
$$= 586 \text{ lbf/in}^2$$

$$\frac{K_u}{f_c'} = \frac{586 \ \frac{\text{lbf}}{\text{in}^2}}{3000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.195$$

From App. 2.A, assuming a tension-controlled section, the corresponding tension reinforcement index is

$$\omega = 0.255$$

The required reinforcement ratio is the lesser of

$$\rho = \frac{\omega f_c'}{f_y} = \frac{(0.255)\left(3000 \ \frac{\text{lbf}}{\text{in}^2}\right)}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.0128$$

The limiting reinforcement ratio for a tension-controlled section is

$$\rho_t = 0.319\beta_1 \frac{f_c'}{f_y}$$
$$= (0.319)(0.85)\left(\frac{3000 \ \frac{\text{lbf}}{\text{in}^2}}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}\right)$$
$$= 0.0136$$
$$> \rho$$

Therefore, the section is tension-controlled.

The minimum allowable reinforcement ratio is the greater of

$$\rho_{\text{min}} = \frac{200 \ \frac{\text{lbf}}{\text{in}^2}}{f_y}$$
$$= \frac{200 \ \frac{\text{lbf}}{\text{in}^2}}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.0033$$
$$< \rho \quad [\text{satisfactory}]$$

$$\rho_{\text{min}} = \frac{3\sqrt{f_c'}}{f_y} = \frac{3\sqrt{3000 \ \frac{\text{lbf}}{\text{in}^2}}}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.00274$$
$$< \rho \quad [\text{satisfactory}]$$

The reinforcement area required is

$$A_s = \rho b d$$
$$= (0.0128)(12 \text{ in})(16 \text{ in})$$
$$= 2.45 \text{ in}^2$$

Beams with Compression Reinforcement

A reinforced concrete beam with compression reinforcement is shown in Fig. 2.2. Compression reinforcement and additional tension reinforcement are required when the factored moment on the member exceeds the design flexural strength of a singly reinforced member with the strain in the tension steel, $\epsilon_t = 0.005$. The residual moment is given by

$$M_r = M_u - M_{\text{max}}$$

The applied strength level moment at midheight of the wall is given by TMS 402 Eq. 9-30 as

$$\begin{aligned} M_{u1} &= \frac{w_u h^2}{8} + \frac{P_{uf}e_u}{2} + P_u\delta_{u1} \\ &= \frac{\left(0.03\ \frac{\text{kip}}{\text{ft}}\right)(20\ \text{ft})^2}{8} + \frac{(0.36\ \text{kip})(7.5\ \text{in})}{(2)\left(12\ \frac{\text{in}}{\text{ft}}\right)} \\ &\quad + \frac{(1.04\ \text{kips})(0.20\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 1.63\ \text{ft-kips} \\ &< \phi M_n \quad \text{[satisfactory]} \\ &< M_{cr} \quad \text{[TMS 402 Eq. 9-30 applies]} \end{aligned}$$

The deflection corresponding to the factored moment is determined in accordance with TMS 402 Sec. 9.3.5.4.2. The moment of inertia of the net wall section is

$$\begin{aligned} I_n &= \frac{bt^3}{12} \\ &= \frac{(1\ \text{ft})(7.63\ \text{in})^3\left(12\ \frac{\text{in}}{\text{ft}}\right)}{12} \\ &= 444\ \text{in}^4 \end{aligned}$$

The modulus of elasticity of reinforcement is given by TMS 402 Sec. 4.2.2.1 as

$$E_s = 29{,}000\ \text{kips/in}^2$$

The modulus of elasticity of concrete masonry is given by TMS 402 Sec. 4.2.2.2.1 as

$$\begin{aligned} E_m &= 900 f'_m \\ &= (900)\left(\frac{1500\ \frac{\text{lbf}}{\text{in}^2}}{1000\ \frac{\text{lbf}}{\text{kip}}}\right) \\ &= 1350\ \text{kips/in}^2 \end{aligned}$$

The modular ratio is

$$\begin{aligned} n &= \frac{E_s}{E_m} = \frac{29{,}000\ \frac{\text{kips}}{\text{in}^2}}{1350\ \frac{\text{kips}}{\text{in}^2}} \\ &= 21.5 \end{aligned}$$

The distance from the extreme compression fiber to the neutral axis is given by TMS 402 Eq. 9-35 as

$$\begin{aligned} c &= \frac{A_s f_y + P_u}{0.64 f'_m b} \\ &= \frac{\left((0.15\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right) + 1.04\ \text{kips}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)}{(0.64)\left(1500\ \frac{\text{lbf}}{\text{in}^2}\right)(12\ \text{in})} \\ &= 0.87\ \text{in} \end{aligned}$$

Allowing for the axial load, the moment of inertia of the cracked transformed section about the neutral axis is given by TMS 402 Eq. 9-34 as

f_y

The f_y should be the denominator under P_u

$$\begin{aligned} I_{cr} &= \frac{bc^3}{3} + n\left(\frac{A_s + P_u}{\not{f_y}}\right)(d - c)^2 \\ &= \frac{(1\ \text{ft})(0.87\ \text{in})^3\left(12\ \frac{\text{in}}{\text{ft}}\right)}{3} \\ &\quad + (21.5)\left(\frac{0.15\ \text{in}^2 + 1.04\ \text{kips}}{\not{60}\ \frac{\not{\text{kips}}}{\not{\text{in}}^2}}\right) \\ &\quad \times\left(\frac{7.63\ \text{in}}{2} - 0.87\ \text{in}\right)^2 \\ &= 33.8\ \text{in}^4 \end{aligned}$$

add 60 kips/in^2 as the denominator under just the 1.04 kips

Since $M_{u1} < M_{cr}$, the midheight deflection corresponding to the factored moment is derived from TMS 402 Eq. 9-29 as

$$\begin{aligned} \delta_u &= \frac{5M_{cr}h^2}{48E_mI_n} \\ &= \frac{(5)(1.69\ \text{ft-kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)\left((20\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)\right)^2}{(48)\left(1350\ \frac{\text{kips}}{\text{in}^2}\right)(444\ \text{in}^4)} \\ &= 0.203\ \text{in} \end{aligned}$$

[This is approximately equal to the assumed deflection; further iterations are unnecessary.]

So,

$$\begin{aligned} M_u &= 1.63\ \text{ft-kips} \\ &< \phi M_n \quad \text{[satisfactory]} \end{aligned}$$

The flexural capacity is adequate.

Reinforced Masonry