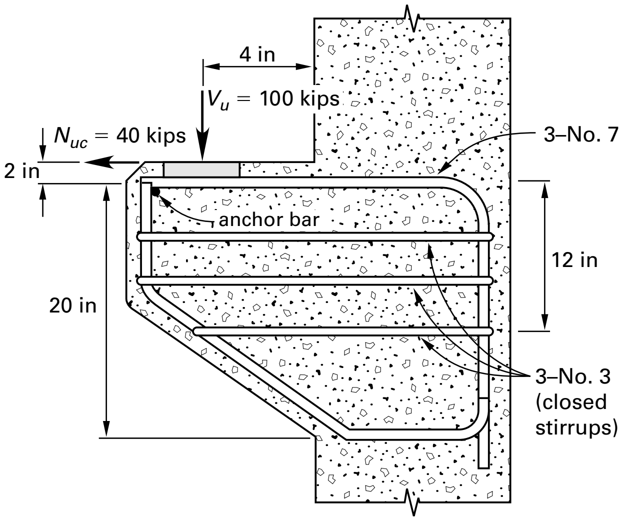
concrete compressive strength of 3000 lbf/in^2. Determine whether the corbel is adequate for the applied factored loads indicated.



Solution

$$\begin{aligned}0.2\phi f_c' b_w d &= (0.2)(0.75)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(15\text{ in})(20\text{ in})\\ &= 135\text{ kips}\\ &> V_u\\ (480 + 0.08f_c')\phi b_w d &= (0.72)(0.75)(15\text{ in})(20\text{ in})\\ &= 162\text{ kips}\\ &> V_u\\ 1.6\phi b_w d &= (1.6)(0.75)(15\text{ in})(20\text{ in})\\ &= 360\text{ kips}\\ &> V_u\end{aligned}$$

The corbel conforms to ACI Table 22.9.4.4.

The shear friction reinforcement area is given by ACI Sec. 22.9.~~9~~.2 as

$$\begin{aligned}A_{vf} = \frac{V_u}{\phi f_y \mu} &= \frac{100\text{ kips}}{(0.75)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)(1.4)}\\ &= 1.59\text{ in}^2\end{aligned}$$

The tension reinforcement area required is

$$\begin{aligned}A_n = \frac{N_{uc}}{\phi f_y} &= \frac{40\text{ kips}}{(0.75)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}\\ &= 0.889\text{ in}^2\end{aligned}$$

The factored moment acting on the corbel is

$$\begin{aligned}M_u &= V_u a_v + N_{uc}(h-d)\\ &= (100\text{ kips})(4\text{ in}) + (40\text{ kips})(2\text{ in})\\ &= 480\text{ in-kips}\end{aligned}$$

The area of flexural reinforcement required for $\phi = 0.75$ for corbels as given by ACI Sec. R21.2.1 for corbels is

$$\begin{aligned}A_f &= \frac{0.85bdf_c'\left(1-\sqrt{1-\frac{M_u}{0.319b_w d^2 f_c'}}\right)}{f_y}\\ &= 0.545\text{ in}^2\end{aligned}$$

The primary reinforcement area required is given by ACI Sec. 16.5.5.1 as

$$\begin{aligned}A_{sc} &= A_f + A_n\\ &= 0.545\text{ in}^2 + 0.889\text{ in}^2\\ &= 1.434\text{ in}^2\end{aligned}$$

Three no. 7 bars are provided, giving an area of

$$\begin{aligned}A_s' &= 1.80\text{ in}^2\\ &> 1.434\text{ in}^2\quad\text{[satisfactory]}\end{aligned}$$

Also, from ACI Sec. 16.5.5.1, the area of primary reinforcement must not be less than

$$\begin{aligned}\frac{2A_{vf}}{3} + A_n &= \frac{(2)(1.59\text{ in}^2)}{3} + 0.889\text{ in}^2\\ &= 1.95\text{ in}^2\\ &> A_s'\quad\text{[unsatisfactory]}\end{aligned}$$

The area of closed stirrups required is given by ACI Sec. 16.5.5.2 as

$$\begin{aligned}A_h &= \frac{A_{sc} - A_n}{2}\\ &= \frac{1.95\text{ in}^2 - 0.889\text{ in}^2}{2}\\ &= 0.53\text{ in}^2\end{aligned}$$

Three no. 3 closed stirrups are provided, giving an area of

$$\begin{aligned}A_h &= 0.66\text{ in}^2\\ &> 0.53\text{ in}^2\quad\text{[satisfactory]}\end{aligned}$$

The development length for a grade 60, no. 8 bar, with 2.5 in side cover and 2 in end cover and with a standard 90° hook, is given by ACI Sec. 25.4.3.1 as

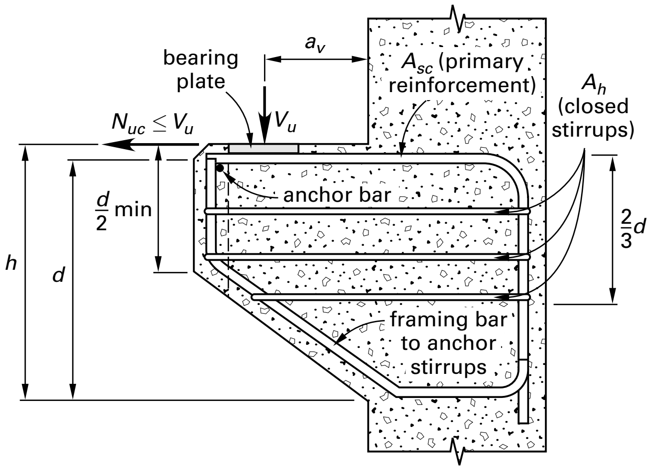
$$\begin{aligned} l_{dh} &= \frac{(0.7)\left(1200\ \frac{\text{lbf}}{\text{in}^2}\right)d_b}{\sqrt{f_c'}} \\ &= \frac{(0.7)\left(1200\ \frac{\text{lbf}}{\text{in}^2}\right)(1\ \text{in})}{\sqrt{4500\ \frac{\text{lbf}}{\text{in}^2}}} \\ &= 12.5\ \text{in} \\ &> l_a \quad \text{[Anchorage length is inadequate.]} \end{aligned}$$

Use an end plate to anchor the bars.

8. CORBELS

A corbel is a cantilever bracket supporting a load-bearing member. As shown in Fig. 2.14 and specified in ACI Sec. 16.5.1.1, the shear span-to-depth ratio, a_v/d, and the ratio of horizontal tensile force to vertical force, N_{uc}/V_u, are limited to a maximum value of unity. The depth of the corbel at the outside edge of bearing area must not be less than $d/2$.

Figure 2.14 *Corbel Details*



At the face of the support, the forces acting on the corbel are a shear force, V_u, a moment $(V_u a_v + N_{uc}(h - d))$, and a tensile force, N_{uc}. These require reinforcement areas of A_{vf}, A_f, and A_n, respectively. The shear friction reinforcement area is derived from ACI Eq. 22.9.4.2 as

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

Also, from ACI Table 22.9.4.4, the factored shear force on the section is

$$\begin{aligned} V_u &\le 0.2\phi f_c' b_w d \\ &\le (480 + 0.08 f_c')\phi b_w d \\ &\le 1600\phi b_w d \end{aligned}$$

$$\begin{aligned} \mu &= \text{coefficient of friction at face of support,} \\ &\quad \text{as given by ACI 22.9.4.2} \\ &= 1.4\lambda \quad \text{[for concrete placed monolithically]} \end{aligned}$$

The correction factor related to the unit weight of concrete is defined by ACI Sec. 19.2.4.2 as

$$\begin{aligned} \lambda &= 1.0 \quad \text{[for normal weight concrete]} \\ &= 0.75 \quad \text{[for all lightweight concrete]} \end{aligned}$$

In accordance with ACI Sec. 16.5.3.5, the tensile force N_{uc} may not be less than $0.2V_u$, and the corresponding area of reinforcement required is

$$A_n = \frac{N_{uc}}{\phi f_y}$$

The required area of primary tension reinforcement is given by ACI Sec. 16.5.5.1 as the greater of

$$A_{sc} = A_f + A$$

$$\frac{A_{sc}}{bd} = \frac{0.04 f_c'}{f_y}$$

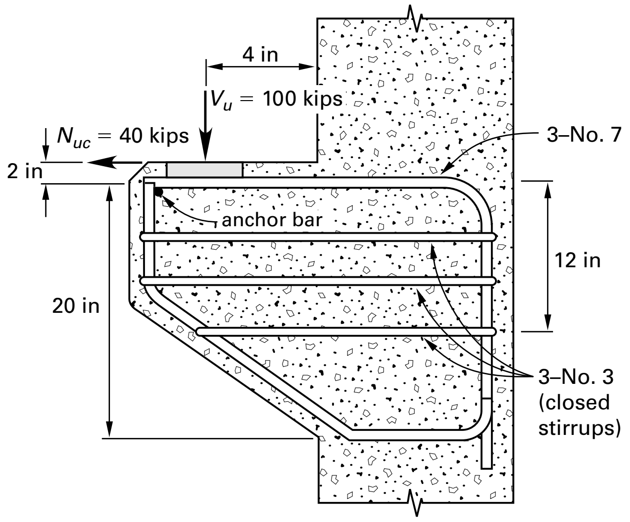
The minimum required area of closed ties distributed over a depth of $2d/3$ is given by ACI Sec. 16.5.5.2 as

$$A_h = \frac{A_{sc} - A_n}{2}$$

ACI Table 21.2.1 gives the value of the strength reduction factor for corbels as $\phi = 0.75$.

Example 2.12

The reinforced concrete corbel shown has a width of 15 in, is reinforced with grade 60 bars, and has a concrete compressive strength of 3000 lbf/in^2. Determine whether the corbel is adequate for the applied factored loads indicated.



Solution

$$\begin{aligned}
0.2\phi f_c' b_w d &= (0.2)(0.75)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(15\ \text{in})(20\ \text{in}) \\
&= 135\ \text{kips} \\
&> V_u \\
(480 + 0.08 f_c')\phi b_w d &= (0.72)(0.75)(15\ \text{in})(20\ \text{in}) \\
&= 162\ \text{kips} \\
&> V_u \\
1.6\phi b_w d &= (1.6)(0.75)(15\ \text{in})(20\ \text{in}) \\
&= 360\ \text{kips} \\
&> V_u
\end{aligned}$$

The corbel conforms to ACI Table 22.9.4.4.

The shear friction reinforcement area is given by ACI Sec. 22.9.9.2 as

$$\begin{aligned}
A_{vf} = \frac{V_u}{\phi f_y \mu} &= \frac{100\ \text{kips}}{(0.75)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)(1.4)} \\
&= 1.59\ \text{in}^2
\end{aligned}$$

The tension reinforcement area required is

$$\begin{aligned}
A_n = \frac{N_{uc}}{\phi f_y} &= \frac{40\ \text{kips}}{(0.75)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)} \\
&= 0.889\ \text{in}^2
\end{aligned}$$

The factored moment acting on the corbel is

$$\begin{aligned}
M_u &= V_u a_v + N_{uc}(h - d) \\
&= (100\ \text{kips})(4\ \text{in}) + (40\ \text{kips})(2\ \text{in}) \\
&= 480\ \text{in-kips}
\end{aligned}$$

The area of flexural reinforcement required for $\phi = 0.75$ for corbels as given by ACI Sec. R21.2.1 for corbels is

$$\begin{aligned}
A_f &= \frac{0.85 b d f_c' \left(1 - \sqrt{1 - \dfrac{M_u}{0.319 b_w d^2 f_c'}}\right)}{f_y} \\
&= 0.545\ \text{in}^2
\end{aligned}$$

The primary reinforcement area required is given by ACI Sec. 16.5.5.1 as

$$\begin{aligned}
A_{sc} &= A_f + A_n \\
&= 0.545\ \text{in}^2 + 0.889\ \text{in}^2 \\
&= 1.434\ \text{in}^2
\end{aligned}$$

Three no. 7 bars are provided, giving an area of

$$\begin{aligned}
A_s' &= 1.80\ \text{in}^2 \\
&> 1.434\ \text{in}^2 \quad \text{[satisfactory]}
\end{aligned}$$

Also, from ACI Sec. 16.5.5.1, the area of primary reinforcement must not be less than

$$\begin{aligned}
\boxed{\frac{2A_{vf}}{3} + A_n} &= \frac{(2)(1.59\ \text{in}^2)}{3} + 0.889\ \text{in}^2 \\
&= 1.95\ \text{in}^2 \\
&> A_s' \quad \text{[unsatisfactory]}
\end{aligned}$$

The area of closed stirrups required is given by ACI Sec. 16.5.5.2 as

$$\begin{aligned}
A_h &= \frac{A_{sc} - A_n}{2} \\
&= \frac{1.95\ \text{in}^2 - 0.889\ \text{in}^2}{2} \\
&= 0.53\ \text{in}^2
\end{aligned}$$

Three no. 3 closed stirrups are provided, giving an area of

$$\begin{aligned}
A_h &= 0.66\ \text{in}^2 \\
&> 0.53\ \text{in}^2 \quad \text{[satisfactory]}
\end{aligned}$$

Eleven stress categories are defined in AISC 360 Table A-3.1. For stress categories A, B, B′, C, D, E, and E′, the design stress range in the member must not exceed the value given by AISC 360 Eq. A-3-1 as

$$\begin{aligned} F_{SR} &= \left(\frac{C_f}{n_{SR}}\right)^{0.333} \\ &\geq F_{TH} \end{aligned}$$

For stress category F, the design stress range in the member must not exceed the value given by AISC 360 Eq. A-3-2.

$$\begin{aligned} F_{SR} &= \left(\frac{C_f}{n_{SR}}\right)^{0.167} \\ &\geq F_{TH} \end{aligned}$$

Example 5.34

A tie member in a steel truss consists of a pair of grade A36 5 in × 5 in × ⅜ in angles fillet welded to a gusset plate. The force in the member, due to dead load only, is 90 kips tension. The additional force in the member, due to live load only, varies from a compression of 7 kips to a tension of 50 kips. During the design life of the structure, the live load may be applied 600,000 times. Determine whether fatigue effects are a concern.

Solution

From AISC 360 Table A-3.1, the loading condition of Sec. 4.1 is applicable and the relevant factors are

$$\begin{aligned} E &= \text{stress category} \\ F_{SR} &= \text{allowable stress range} \\ &= \left(\frac{C_f}{n_{SR}}\right)^{0.333} = \left(\frac{11\times 10^8}{6\times 10^5}\right)^{0.333} \\ &= 12.21\ \text{kips/in}^2 \end{aligned}$$

The area of the tie is

$A_s =$ ~~7.22~~ in^2

The actual stress range is

$f_{SR} = \dfrac{T_{\max} - T_{\min}}{A_s} = \dfrac{50\ \text{kips} - (-7\ \text{kips})}{\text{~~7.22~~ in}^2}$

$=$ ~~7.9~~ kips/in^2 [exceeds F_{TH}]

$< F_{SR}$

This is within the allowable stress range, so fatigue effects ~~need not be considered~~.

8. DESIGN OF BOLTED CONNECTIONS

Nomenclature

A_b	nominal unthreaded body area of bolt	in^2
C	coefficient for eccentrically loaded bolt and weld groups	–
d	nominal bolt diameter	in
d_m	moment arm between resultant tensile and compressive forces due to an eccentric force	in
d_n	nominal hole diameter	–
D_u	a multiplier that reflects the ratio of the mean installed bolt tension to the specified minimum bolt pretension	–
f_{rv}	required shear stress	kips/in^2
f_v	computed shear stress	kips/in^2
F_{nt}	nominal tensile stress of bolt	kips/in^2
F'_{nt}	nominal tensile stress of a bolt subjected to combined shear and tension	kips/in^2
F_{nv}	nominal shear stress of bolt	kips/in^2
F_u	specified minimum tensile strength	kips/in^2
h_{sc}	modification factor for type of hole	–
k_s	slip-critical combined tension and shear coefficient	–
l_c	clear distance, in the direction of force, between the edge of the hole and the edge of the adjacent hole or edge of the material	in
L_e	edge distance between the bolt center and the edge of the connected part	in
n	number of bolts in a connection	–
n'	number of bolts above the neutral axis (in tension)	–
n_b	number of bolts carrying strength level tension T_u	–
n_s	number of slip planes	–
P_r	load on connection	kips
R_n	nominal strength	kips
s	center-to-center pitch of two consecutive bolts	in
t	thickness of connected part	in
T_a	applied tensile force (ASD)	kips
T_b	minimum pre-tension force	kips
T_u	applied tensile force (LRFD)	kips

Symbols

μ	mean slip coefficient for the applicable surface	–
ϕ	resistance factor	–

Types of Bolts

There are two categories of bolts: common bolts and high-strength bolts. High-strength bolts are additionally grouped by strength levels into two categories: group A bolts (A325, F1852, A354 grade BC, and A449) and group B bolts (A490, F2280, and A354 grade BD).

Common bolts of grade A307 with a nominal tensile strength of 45 kips/in^2 are used in snug-tight (bearing-type) connections only.

High-strength bolts in group A with a nominal tensile strength of 90 kips/in^2, or group B with a nominal tensile strength of 113 kips/in^2, are used in bearing-type, pretensioned, and slip-critical connections.

Bolts are installed in the following three types of connections.

1. *Bearing-type* or *snug-tight connections* require the bolts to be tightened sufficiently to bring the plies into firm contact. Levels of installed tension are not specified. Transfer of the load from one connected part to another depends on the bearing of the bolts against the side of the holes. This type may be used when pretensioned or slip-critical connections are not required.
2. *Pretensioned connections* require the bolts to be pretensioned to a minimum value of 70% of the bolt's minimum tensile strength and the faying surfaces may be uncoated, coated, or galvanized regardless of the slip coefficient. Transfer of the load from one connected part to another depends on the bearing of the bolts against the side of the holes. Pretensioned connections are required when bearing-type connections are used in
    - column splices in buildings over 125 ft in height
    - bracing members in buildings over 125 ft in height
    - structures carrying cranes of over 5 ton capacity
    - supports of machinery causing impact or stress reversal
3. *Slip-critical connections* require the bolts to be pretensioned to a minimum value of 70% of the bolt's tensile strength, and the faying surfaces must be prepared to produce a specific value of the slip coefficient. Transfer of the load from one connected part to another depends on the friction induced between the parts. Slip-critical connections are required where
    - fatigue load occurs
    - bolts are used in oversize holes or slotted holes parallel to the direction of load
    - slip at the faying surfaces will affect the performance of the structure
    - bolts are used in conjunction with welds

Bearing-Type Bolts in Shear

The minimum permissible distance and the preferred distance between the centers of holes is given by AISC 360 Sec. J3.3 as

$$s_{\text{min}} = 2.67d$$
$$s_{\text{pref}} = 3.0d$$

The nominal shear strength is based on the nominal unthreaded cross-sectional area of the bolt, A_b, and the nominal shear strength, F_{nv}. Nominal shear strength of fasteners and threaded parts is given in AISC 360 Table J3.2, and for high-strength bolts a reduced nominal strength is applicable when threads are not excluded from the shear planes. No reduction is made for A307 bolts. For connections longer than 38 in, the nominal strength is reduced. The bolt's available shear capacity is obtained from AISC 360 Eq. J3-1.

$$\begin{aligned}\phi R_n &= \phi F_{nv} A_b \\ &= 0.75 F_{nv} A_b \quad \text{[LRFD]} \\ \frac{R_n}{\Omega} &= \frac{F_{nv} A_b}{\Omega} \\ &= \frac{F_{nv} A_b}{2.00} \quad \text{[ASD]}\end{aligned}$$

Bearing-Type Bolts in Tension and Combined Shear and Tension

The available strength in tension is given by AISC 360 Sec. J3.6 as

$$\begin{aligned}\phi R_n &= \phi F_{nt} A_b \\ &= 0.75 F_{nt} A_b \quad \text{[LRFD]} \\ \frac{R_n}{\Omega} &= \frac{F_{nt} A_b}{\Omega} \\ &= \frac{F_{nt} A_b}{2.00} \quad \text{[ASD]}\end{aligned}$$

Values of the nominal tensile stress F_{nt} are given in AISC 360 Table J3.2 for all types of bolts. Values of ϕR_n and R_n/Ω are given in *AISC Manual* Table 7-2.

When a bearing-type bolt is subjected to combined shear and tension, the available strength in shear is unaffected, and the available strength in tension is reduced in accordance with AISC 360 Sec. J3.7.

The value of the reduced available tensile capacity is

$$\begin{aligned}\phi R_n &= \phi F'_{nt} A_b \\ &= 0.75 F'_{nt} A_b \quad \text{[LRFD]} \\ \frac{R_n}{\Omega} &= \frac{F'_{nt} A_b}{\Omega} \\ &= \frac{F'_{nt} A_b}{2.00} \quad \text{[ASD]}\end{aligned}$$

F'_{nt}, the modified nominal tensile stress, is calculated using AISC 360 Eq. J3-3a for the LRFD method or AISC 360 Eq. J3-3b for the ASD method.

$$\begin{aligned}F'_{nt} &= 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \quad \text{[LRFD]} \\ &= 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \quad \text{[ASD]}\end{aligned}$$

The required shear stress, f_{rv}, is determined using appropriate load combinations, and must be equal to or less than the available shear stress. Values of the nominal shear stress F_{nv} are given in AISC 360 Table J3.2 for all types of bolts.

When either $f_{rv} \le 30\%$ of the available shear stress or $f_t \le 30\%$ of the available tensile stress, the effects of combined stress do not need to be considered.

Example 5.35

The connection analyzed in Ex. 5.32 consists of 11 grade A307 $\frac{7}{8}$ in diameter bolts. Determine the design shear strength of the bolts in the connection.

Solution

From *AISC Manual* Table 7-1, the available strength of the 11 bolts in single shear is

LRFD Method

$$\begin{aligned}\phi R_n &= \phi F_{nv} A_b n \\ &= \left(12.2\ \frac{\text{kips}}{\text{bolt}}\right)(11\ \text{bolts}) \\ &= 134\ \text{kips}\end{aligned}$$

ASD Method

$$\begin{aligned}\frac{R_n}{\Omega} &= \frac{F_{nv} A_b n}{\Omega} \\ &= \left(8.11\ \frac{\text{kips}}{\text{bolt}}\right)(11\ \text{bolts}) \\ &= 89\ \text{kips}\end{aligned}$$

Slip-Critical Bolts in Shear

Slip-critical bolts are high strength group A or group B bolt pretensioned to the value specified in AISC 360 Table J3.1 of

$$T_b = 0.70F_u$$

The pretension produces a clamping force between the parts, and transfers the shear load from one connected part to another by friction. At the strength limit state, the connection may slip sufficiently to place the bolts in bearing and AISC 360 Sec. J3.10 requires slip-critical connections to also comply with the requirements of snug-tight connections.

The *frictional resistance* developed in a slip-critical connection depends on the condition of the faying surfaces. The values of the slip coefficient, μ, for two types of surface conditions (class A and class B) are given in AISC 360 Sec. J3.8.

- Class A surface conditions consist of unpainted clean mill scale surfaces or blast-cleaned surfaces with class A coatings. The slip coefficient is

$$\mu = 0.30$$

- Class B surface conditions consist of unpainted blast-cleaned surfaces or blast-cleaned surfaces with class B coatings. The slip coefficient is

$$\mu = 0.50$$

The *nominal slip resistance* is identical for the cases of threads included or excluded from the shear plane, and is given by AISC 360 Eq. J3-4 as

$$R_n = \mu D_u h_f T_b n_s$$

The *bolt tension multiplier* reflects the ratio of the mean installed bolt tension to the specified minimum bolt pretension and is given by

$$D_u = 1.13$$

The *modification factor for fillers*, h_f, is

- 1.00 where bolts are added to distribute loads in the filler
- 1.00 for one filler between connected parts
- 0.85 for two or more fillers between connected parts

The resistance factor and safety factor adopted depend on the type of hole used in the connection. Connections allowing a large amount of slip may cause unacceptable slip and will require a higher safety factor than a