The temperature of the gases in the furnace is 1800°F. The heat transfer coefficient on the inside of the furnace is 37 Btu/hr-ft²-°F. and the heat transfer coefficient for ambient air outside the furnace is 5 Btu/hr-ft²-°F. The ambient air temperature is 70°F. The overall heat transfer coefficient for the furnace is most nearly

(A) 0.104 Btu/hr-ft²-°F

(B) 0.213 Btu/hr-ft²-°F

(C) 0.345 Btu/hr-ft²-°F

(D) 0.467 Btu/hr-ft²-°F

Solution

The overall resistance to heat transfer per unit area is obtained by adding up the individual resistances per unit area. The overall heat transfer coefficient for the furnace is

$$\frac{1}{U} = \frac{1}{h_\text{i}} + \left(\frac{t}{k}\right)_\text{b} + \left(\frac{t}{k}\right)_\text{s} + \left(\frac{t}{k}\right)_\text{i} + \frac{1}{h_\text{o}}$$

$$U = \frac{1}{\frac{1}{h_\text{i}} + \left(\frac{t}{k}\right)_b + \left(\frac{t}{k}\right)_\text{s} + \left(\frac{t}{k}\right)_\text{i} + \frac{1}{h_\text{o}}}$$

$$= \frac{1}{\begin{array}{l}\dfrac{1}{37\ \dfrac{\text{Btu}}{\text{hr-ft}^2\text{-°F}}} + \dfrac{6 \text{ in}}{\left(0.41\ \dfrac{\text{Btu}}{\text{hr-ft-°F}}\right)\left(12\ \dfrac{\text{in}}{\text{ft}}\right)} \\ + \dfrac{2 \text{ in}}{\left(25\ \dfrac{\text{Btu}}{\text{hr-ft-°F}}\right)\left(12\ \dfrac{\text{in}}{\text{ft}}\right)} \\ + \dfrac{3 \text{ in}}{\left(0.03\ \dfrac{\text{Btu}}{\text{hr-ft-°F}}\right)\left(12\ \dfrac{\text{in}}{\text{ft}}\right)} \\ + \dfrac{1}{\left(5\ \dfrac{\text{Btu}}{\text{hr-ft}^2\text{-°F}}\right)\left(12\ \dfrac{\text{in}}{\text{ft}}\right)}\end{array}}$$

$$= 0.1041 \text{ Btu/hr-ft}^2\text{-°F} \quad (0.104 \text{ Btu/hr-ft}^2\text{-°F})$$

The answer is (A).

When conduction and convection (but not radiation) are the only modes of heat transfer, the *overall coefficient of heat transfer*, U, also known as the *overall conductivity*, *overall heat transmittance*, or simply *U-factor*, is defined by Eq. 17.32.

$$U = \frac{1}{AR_\text{th,total}} = \frac{1}{\sum_i \frac{L_i}{k_i} + \sum_j \frac{1}{h_j}} \qquad 17.32$$

The overall coefficient is usually used in conjunction with the outside (exposed) surface area (because it is easier to measure), but not always. Therefore, it will generally be written with a subscript (e.g., U_o or U_i) indicating which area it is based on. The heat transfer is

$$Q = U_o A_o (T_1 - T_2) = U_i A_i (T_1 - T_2) \qquad 17.33$$

When heat transfer occurs by two or three modes, the overall coefficient of heat transfer takes all active modes—conduction, convection, and radiation—into consideration. In that sense, the overall coefficient, U, is defined by Eq. 17.34, rather than being used in it.

$$U = \frac{Q}{A(T_1 - T_2)} \qquad 17.34$$

Example 17.4

A 100 ft² (9.3 m²) wall consists of 4 in (10 cm) of red brick (k = 0.38 Btu-ft/hr-ft²-°F (0.66 W/m·K)), 1 in (2.5 cm) of pine (k = 0.06 Btu-ft/hr-ft²-°F (0.10 W/m·K)), and ½ in (1.2 cm) of plasterboard (k = 0.30 Btu-ft/hr-ft²-°F (0.52 W/m·K)). The internal and external film coefficients are 1.65 Btu/hr-ft²-°F and 6.00 Btu/hr-ft²-°F (9.38 W/m²·K and 34.1 W/m²·K), respectively. The inside and outside air temperatures are 72°F (22°C) and 30°F (−1°C), respectively. Determine the heat transfer.

SI Solution

Convert the thicknesses from centimeters to meters.

$$L_\text{brick} = \frac{10 \text{ cm}}{100\ \frac{\text{cm}}{\text{m}}} = 0.10 \text{ m}$$

$$L_\text{pine} = \frac{2.5 \text{ cm}}{100\ \frac{\text{cm}}{\text{m}}} = 0.025 \text{ m}$$

$$L_\text{plasterboard} = \frac{1.2 \text{ cm}}{100\ \frac{\text{cm}}{\text{m}}} = 0.012 \text{ m}$$

Heat Transfer