If A, B, and C are subsets of the universal set, the following laws apply.

- *identity laws*

$$A \cup \varnothing = A \tag{11.1}$$

$$A \cup U = U \tag{11.2}$$

$$A \cap \varnothing = \varnothing \tag{11.3}$$

$$A \cap U = A \tag{11.4}$$

- *idempotent laws*

$$A \cup A = A \tag{11.5}$$

$$A \cap A = A \tag{11.6}$$

- *complement laws*

$$A \cup A' = U \tag{11.7}$$

$$(A')' = A \tag{11.8}$$

$$A \cap A' = \varnothing \tag{11.9}$$

$$U' = \varnothing \tag{11.10}$$

- *commutative laws*

$$A \cup B = B \cup A \tag{11.11}$$

$$A \cap B = B \cap A \tag{11.12}$$

- *associative laws*

$$(A \cup B) \cup C = A \cup (B \cup C) \tag{11.13}$$

$$(A \cap B) \cap C = A \cap (B \cap C) \tag{11.14}$$

- *distributive laws*

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \tag{11.15}$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \tag{11.16}$$

- *de Morgan's laws*

$$(A \cup B)' = A' \cap B' \tag{11.17}$$

$$(A \cap B)' = A' \cup B' \tag{11.18}$$

2. COMBINATIONS OF ELEMENTS

There are a finite number of ways in which n elements can be combined into distinctly different groups of r items. For example, suppose a farmer has a hen, a rooster, a duck, and a cage that holds only two birds. The possible *combinations* of three birds taken two at a time are (hen, rooster), (hen, duck), and (rooster, duck). The birds in the cage will not remain stationary, and the combination (rooster, hen) is not distinctly different from (hen, rooster). That is, the groups are not *order conscious.*

The number of combinations of n items taken r at a time is written $C(n,r)$, C_r^n, ${}_nC_r$, or $\binom{n}{r}$ (pronounced "n choose r") and given by Eq. 11.19. It is sometimes referred to as the *binomial coefficient.*

$$\binom{n}{r} = C(n,r) = \frac{n!}{(n-r)!\,r!} \quad [\text{for } r \leq n] \tag{11.19}$$

Example 11.1

Six people are on a sinking yacht. There are four life jackets. How many combinations of survivors are there?

Solution

The groups are not order conscious. From Eq. 11.19,

$$C(6,4) = \frac{n!}{(n-r)!\,r!} = \frac{6!}{(6-4)!\,4!} = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{(2\cdot 1)(4\cdot 3\cdot 2\cdot 1)}$$
$$= 15$$

3. PERMUTATIONS

An order-conscious subset of r items taken from a set of n items is the *permutation* $P(n,r)$, also written P_r^n and ${}_nP_r$. The permutation is order conscious because the arrangement of two items (say a_i and b_i) as a_ib_i is different from the arrangement b_ia_i. The number of permutations is

$$P(n,r) = \frac{n!}{(n-r)!} \quad [\text{for } r \leq n] \tag{11.20}$$

If groups of the entire set of n items are being enumerated, the number of permutations of n items taken n at a time is

$$P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! \tag{11.21}$$

A *ring permutation* is a special case of n items taken n at a time. There is no identifiable beginning or end, and the number of permutations is divided by n.

$$P_{\text{ring}}(n,n) = \frac{P(n,n)}{n} = (n-1)! \tag{11.22}$$

Example 11.2

A pianist knows four pieces but will have enough stage time to play only three of them. Pieces played in a different order constitute a different program. How many different programs can be arranged?