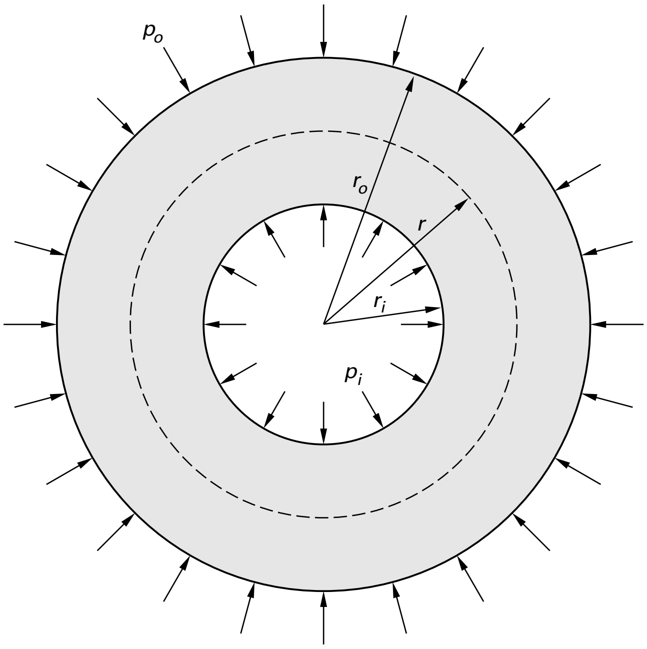
Figure 54.6 *Cylindrical Vessel Subjected to Internal and External Pressures*



Cylindrical Pressure Vessel

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + \dfrac{r_i^2 r_o^2 (p_o - p_i)}{r^2}}{r_o^2 - r_i^2} \qquad 54.2$$

For a thick-walled vessel, the axial stress is given by Eq. 54.3. Axial stress is constant within the wall; it does not vary with the distance from the center of the vessel.

Cylindrical Pressure Vessel

$$\sigma_a = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} \qquad 54.3$$

Equation 54.1 through Eq. 54.3 can often be simplified for specific conditions. When calculating stress on the inner surface, r_i is equal to r; for stress on the outer surface, r_o is equal to r. If the stress is based only on internal pressure, $\cancel{r}_o$ is zero; if the stress is based only on external pressure, p_i is zero. For example, in calculating the tangential stress on the inner surface of a cylindrical vessel ($r_i = r$) due to internal pressure ($\cancel{r}_o = 0$), Eq. ~~54.3~~ can be simplified to Eq. 54.4.

Cylindrical Pressure Vessel

$$\sigma_t = p_i \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \right) \qquad 54.4$$

For a thin-walled cylindrical vessel, tangential and axial stresses vary only with the vessel's diameter, D, and the internal pressure, p_i. Tangential stress can be found from Eq. 54.5, and axial stress can be found from Eq. 54.6. Tangential stress is twice axial stress, so it is tangential stress that drives pressure vessel design. In a thin-walled vessel, radial stress is significantly less than both the tangential and axial stresses and is typically neglected.

Cylindrical Pressure Vessel

$$\sigma_t = \frac{p_i D}{2t} \qquad 54.5$$

$$\sigma_l = \frac{p_i D}{4t} \qquad 54.6$$

Equation 54.5 and Eq. 54.6 can also be used to calculate the stresses in circumferential and longitudinal joints[20], but cannot be used to calculate wall thickness. Wall thickness calculations are addressed in Sec. 54.25.

For a thin-walled pipe with a maximum allowable tangential stress of S, the maximum working pressure can be found from the Barlow formula.

Cylindrical Pressure Vessel

$$p = \frac{2St}{D} \qquad 54.7$$

19. CORROSION ALLOWANCE

An optional *corrosion allowance* compensates for any wall thinning expected over the lifetime of the vessel. The BPVC does not provide guidance in determining the allowance. Values will be unique to each situation and must be determined by the user considering vessel duty and corrosiveness of the contents. Generic total corrosion allowances of one-sixth of the required thickness, $^1/_{16}$ in (1.6 mm), and $^1/_8$ in (3.2 mm) are common, as is a corrosion rate of 0.005 in/yr (0.13 mm/yr), but these have no specific basis other than tradition. A corrosion allowance of zero is common for stainless steel and other nonferrous materials not subject to corrosion. An allowance of 1 mm is typical for air receivers where moisture condensation is inevitable. Usually, the maximum corrosion allowance for carbon steel is 6 mm.

Every dimension used in a formula should be a corroded dimension.[21] When calculating the thickness from a known pressure, if a formula (such as from Table 54.9 or Table 54.10) used to calculate the theoretical thickness includes a nominal radius term, r, the corrosion allowance should be added to the radius before calculating the theoretical thickness. This will account for the slight

[20]The term girth seam is sometimes used when referring to a circumferential joint.

[21]As illustrated in BPVC Sec. VIII, Div. 1, App. L (e.g., Ex. L.9.2.1), this admonition includes adding the corrosion allowance to the nominal radius as well as to the resulting required thickness. The effect of increasing the radius is small, and many authorities simply add the corrosion allowance to the thickness as a final step.

Chapter 52
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4. INTERMEDIATE COLUMNS

Columns with slenderness ratios less than or equal to the column stress determination factor ($S_r < (S_r)_D$), but that are too long to be short piers, are known as *intermediate columns*.

Intermediate Columns

$$(S_r)_D = \sqrt{\frac{2\pi^2 E}{\cancel{K} S_y}} \tag{52.10}$$

Replace with: "K²"

The critical axial load (the buckling load) is given by Eq. 52.11.

Intermediate Columns

$$P_{cr} = A\left(S_y - \frac{K^2}{E}\left(\frac{S_y S_r}{2\pi}\right)^2\right) \tag{52.11}$$

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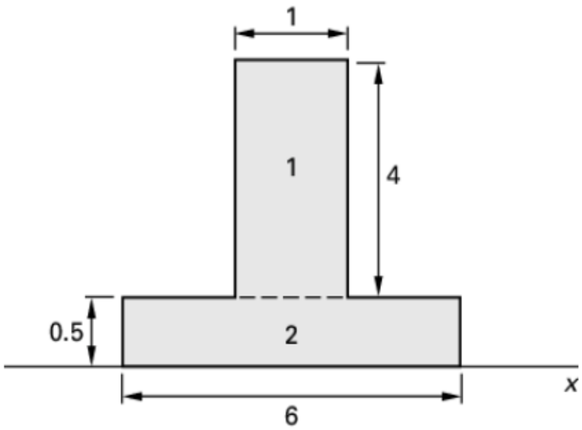
Example 49.4

Find the moment of inertia about the *x*-axis for the inverted-T area shown.



Solution

The area is divided into two basic shapes: 1 and 2. The moment of inertia of basic shape 2 with respect to the *x*-axis is

Properties of Various Shapes

$$I_{x,2} = \frac{bh^3}{3} = \frac{(6.0)\left(0.5\right)^3}{3} = 0.25 \text{ units}^4$$

The moment of inertia of basic shape 1 about its own centroid is

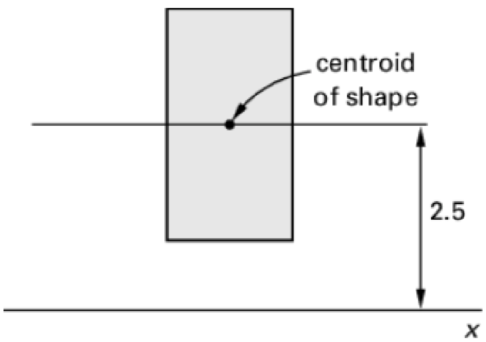
Properties of Various Shapes

$$I_{cx,1} = \frac{bh^3}{12} = \frac{(1)\left(4\right)^3}{12} = 5.33 \text{ units}^4$$

The x-axis is located 2.5 units from the centroid of basic shape 1. From the parallel axis theorem, Eq. 49.20, the moment of inertia of basic shape 1 about the x-axis is

Moment of Inertia Parallel Axis Theorem

$$\begin{aligned} I_{x,1} &= I_{xc,1} + d_1 \overset{\boxed{2}}{A_1^{\cancel{2}}} = 5.33 + (4)\left(2.5\right)^2 \\ &= 30.33 \text{ units}^4 \end{aligned}$$



The total moment of inertia of the T-area is

$$\begin{aligned} I_x &= I_{x,1} + I_{x,2} = 30.33 \text{ units}^4 + 0.25 \text{ units}^4 \\ &= 30.58 \text{ units}^4 \end{aligned}$$

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23. MEASURES OF CENTRAL TENDENCY

It is often unnecessary to present the experimental data in their entirety, either in tabular or graphical form. In such cases, the data and distribution can be represented by various parameters. One type of parameter is a measure of *central tendency*. Mode, median, and mean are measures of central tendency.

The *mode* is the observed value that occurs most frequently. The mode may vary greatly between series of observations. Therefore, its main use is as a quick measure of the central value since little or no computation is required to find it. Beyond this, the usefulness of the mode is limited.

The *median* is the point in the distribution that partitions the total set of observations into two parts containing equal numbers of observations. It is not influenced by the extremity of scores on either side of the distribution. The median is found by counting up (from either end of the frequency distribution) until half of the observations have been accounted for.

Replace with attached eqn

When n is odd, the median is the

$$\left(\cancel{{}^{n}\!/_{2}+1}\right)^{\text{th}}$$

Correction 1

$$\frac{n+1}{2}$$

item value. For even numbers of observations, the median is estimated as some value (i.e., the average) between the two center observations—

$$\left({}^{n}\!/_{2}\right)^{\text{th}} \text{ and } \left({}^{n}\!/_{2}+1\right)^{\text{th}}$$

68 Storage and Disposition of Hazardous Materials

Content in blue refers to the *NCEES Handbook*.

1. GENERAL STORAGE

Storage of *hazardous materials* (*hazmats*) is often governed by local building codes in addition to state and federal regulations. Types of construction, maximum floor areas, and building layout may all be restricted.[1]

Good engineering judgment is called for in areas not specifically governed by the building code. Engineering consideration will need to be given to the following aspects of storage facility design: (1) spill containment provisions, (2) chemical resistance of construction and storage materials, (3) likelihood of and resistance to explosions, (4) exiting, (5) ventilation, (6) electrical design, (7) storage method, (8) personnel emergency equipment, (9) security, and (10) spill cleanup provisions.

2. STORAGE TANKS

Underground storage tanks (USTs) have traditionally been used to store bulk chemicals and petroleum products. Fire and explosion risks are low with USTs, but subsurface pollution is common since inspection is limited. Since 1988, the U.S. Environmental Protection Agency (EPA) has required USTs to have secondary containment, corrosion protection, and leak detection. UST operators also must carry insurance in an amount sufficient to clean up a tank failure.

Because of the cost of complying with UST legislation, above-ground storage tanks (ASTs) are becoming more popular. AST strengths and weaknesses are the reverse of USTs: ASTs reduce pollution caused by leaks, but the expected damage due to fire and explosion is greatly increased. Because of this, some local ordinances prohibit all ASTs for petroleum products.[2]

The following factors should be considered when deciding between USTs and ASTs: (1) space available, (2) zoning ordinances, (3) secondary containment, (4) leak-detection equipment, (5) operating limitations, and (6) economics.

Most ASTs are constructed of carbon or stainless steel. These provide better structural integrity and fire resistance than fiberglass-reinforced plastic and other composite tanks. Tanks can be either field-erected or factory-fabricated (capacities greater than approximately 50,000 gal (190 kL)). Factory-fabricated ASTs are usually designed according to UL-142 (Underwriters Laboratories *Standard for Safety*), which dictates steel type, wall thickness, and characteristics of compartments, bulkheads, and fittings. Most ASTs are not pressurized, but those that are must be designed in accordance with the ASME *Boiler and Pressure Vessel Code*, Section VIII.

NFPA 30 (*Flammable and Combustible Liquids Code*, National Fire Protection Association, Quincy, MA) specifies the minimum separation distances between ASTs, other tanks, structures, and public right-of-ways. The separation is a function of tank type, size, and contents. NFPA 30 also specifies installation, spill control, venting, and testing.

ASTs must be double-walled, concrete-encased, or contained in a dike or vault to prevent leaks and spills, and they must meet fire codes. Dikes should have a capacity in excess (e.g., 110% to 125%) of the tank volume. ASTs (as do USTs) must be equipped with overfill prevention systems. Piping should be above-ground wherever possible. Reasonable protection against vandalism and hunters' bullets is also necessary.[3]

Though they are a good idea, leak-detection systems are not typically required for ASTs. Methodology for leak detection is evolving, but currently includes vacuum or pressure monitoring, electronic gauging, and optical and sniffing sensors. Double-walled tanks may also be fitted with sensors within the interstitial space.

Operationally, ASTs present special problems. In hot weather, volatile substances vaporize and represent an additional leak hazard. In cold weather, viscous content may need to be heated (often by steam tracing).

ASTs are not necessarily less expensive than USTs, but they are generally thought to be so. Additional hidden costs of regulatory compliance, secondary containment, fire protection, and land acquisition must also be considered.

[1]For example, flammable materials stored in rack systems are typically limited to heights of 25 ft (8.3 m).
[2]The American Society of Petroleum Operations Engineers (ASPOE) policy statement states, "Above-ground storage of liquid hydrocarbon motor fuels is inherently less safe than underground storage. Above-ground storage of Class 1 liquids (gasoline) should be prohibited at facilities open to the public."
[3]Approximately 20% of all spills from ASTs are caused by vandalism.

3. DISPOSITION OF HAZARDOUS WASTES

When a hazardous waste is disposed of, it must be taken to a registered *treatment*, *storage*, or *disposal facility* (TSDF). The EPA's *land ban* specifically prohibits the disposal of hazardous wastes on land prior to treatment. Incineration at sea is also prohibited. Waste must be treated to specific maximum concentration limits by specific technology prior to disposal in landfills.

Once treated to specific regulated concentrations, hazardous waste residues can be disposed of by incineration, by landfilling, or, less frequently, by deep-well injection. All disposal facilities must meet detailed design and operational standards.

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Example 50.3

Construct Mohr's circle for Ex. 50.2.

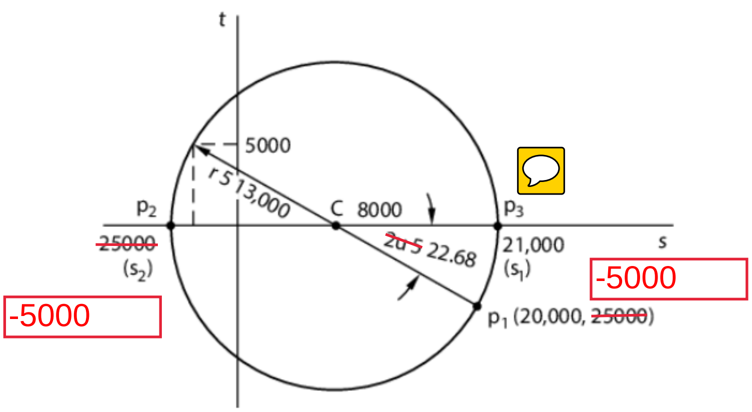
Solution

Mohr's Circle—Stress, 2D

$$
\begin{aligned}
C &= \frac{\sigma_x + \sigma_y}{2} \\
&= \frac{20{,}000\ \frac{\text{lbf}}{\text{in}^2} + \left(-4000\ \frac{\text{lbf}}{\text{in}^2}\right)}{2} \\
&= 8000\ \text{lbf/in}^2
\end{aligned}
$$

Mohr's Circle—Stress, 2D

$$
\begin{aligned}
R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
&= \sqrt{\left(\frac{20{,}000\ \frac{\text{lbf}}{\text{in}^2} - \left(-4000\ \frac{\text{lbf}}{\text{in}^2}\right)}{2}\right)^2 + \left(5000\ \frac{\text{lbf}}{\text{in}^2}\right)^2} \\
&= 13{,}000\ \text{lbf/in}^2
\end{aligned}
$$



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Example 73.16

An asset is purchased for \$9000. Its estimated economic life is 10 years, after which it will be sold for \$200. Find the depreciation in the first three years using straight-line, double declining balance, and sum-of-the-years' digits (SOYD) depreciation methods.

Solution

Use the straight-line method to find the depreciation in the first three years.

Depreciation: Straight Line

$$\begin{aligned} D &= \frac{C - S_n}{n} \\ &= \frac{\$9000 - \$200}{10} \\ &= \$880 \end{aligned}$$

Use the double-declining balance method.

Move to location indicated by the arrow.

$$\begin{aligned} \text{SOYD: } T &= \left(\frac{1}{2}\right)(10)(11) = 55 \\ D_1 &= \left(\frac{10}{55}\right)(\$9000 - \$200) = \$1600 \text{ in year 1} \\ D_2 &= \left(\frac{9}{55}\right)(\$8800) = \$1440 \text{ in year 2} \\ D_3 &= \left(\frac{8}{55}\right)(\$8800) = \$1280 \text{ in year 3} \end{aligned}$$

$$\begin{aligned} D_j &= \frac{2C}{n} \\ D_1 &= \frac{(2)(\$9000)}{10} = \$1800 \\ D_2 &= \frac{(2)(\$9000 - \$1800)}{10} = \$1440 \\ \cancel{D_3} &\cancel{= \frac{(2)(\$9000 - \$200)}{(10)} = \$1152} \end{aligned}$$

Replace with Correction 1

Correction 1

$$D_3 = \frac{(2)(\$9000 - (\$1800 + \$1440))}{10} = \$1152$$

Use the SOYD depreciation method to find the depreciation in the first three years.

Depreciation: Sum-of-Years Digits Method

$$\begin{aligned} D_j &= \frac{2(C - S_n)(n - j + 1)}{n(n+1)} \\ D_1 &= \frac{(2)(\$9000 - \$200)(10 - 1 + 1)}{(10)(10+1)} = \$1600 \\ D_2 &= \frac{(2)(\$9000 - \$200)(10 - 2 + 1)}{(10)(10+1)} = \$1440 \\ D_3 &= \frac{(2)(\$9000 - \$200)(10 - 3 + 1)}{(10)(10+1)} = \$1280 \end{aligned}$$

place the word "or" between sets of equations.

The *helix direction* for single springs can be either right hand or left hand. If the spring works over a threaded member, the winding direction should be opposite of the thread direction. With two *nested springs* (i.e., one spring inside the other), the winding directions must be opposite to prevent intermeshing. Also for nested springs, the outer spring should support approximately $^2/_3$ of the total load. The solid and free lengths of both springs should be approximately the same.

Table 53.4 *Effect of End Treatment on Helical Compression Springs*

	end treatment			
variable	**plain**	**plain and ground**	**squared only**	**squared and ground**
end coils, $\cancel{N}$ N_e	0	1	2	2
total coils, N_t	N	$N+1$	$N+2$	$N+2$
free length, L_0	$pN+d$	$p(N+1)$	$pN+3d$	$pN+2d$
solid length [a], L_s	$d(N_t+1)$	dN_t	$d(N_t+1)$	dN_t
pitch, p	$\frac{L_0-d}{N}$	$\frac{L_0}{N+1}$	$\frac{L_0-3d}{N}$	$\frac{L_0-2d}{N}$

a Use the effective wire diameter when calculating the solid height. This includes the nominal wire diameter, the wire tolerance, and the paint or plating thickness.

6. HELICAL COMPRESSION SPRINGS: DESIGN

Conventional spring design is an iterative procedure. One or more parameters are varied until the requirements are satisfied. Often one or more parameters are unknown and must be assumed to complete the design. For example, when the wire diameter is unknown, the allowable stress and Wahl factor can both only be estimated. (For the initial iteration, it is common to assume a Wahl factor of 1.1.) The outside diameter of the spring may also be a limited factor, as when the spring must fit in a hole.

An important decision is whether the allowable stress is comparable to the maximum working stress or the stress when the spring is compressed solid. Since most helical compression springs will be compressed solid sometime in their lives, it seems logical to use the

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Example 62.19

A factory dispenses candy into individual bags. Three observations of the process are made every 2 hr until 10 overall samples have been taken. The results of those observations are as shown.

sample	observations			$\overline{X}_i$	r_i
1	20	19	21	20.00	2
2	20	20	20	20.00	0
3	21	20	19	20.00	2
4	19	22	20	20.33	3
5	21	20	19	20.00	2
6	20	20	21	20.33	1
7	19	20	21	20.00	2
8	20	22	19	20.33	3
9	19	20	22	20.33	3
10	20	21	20	20.33	1

Determine the X-chart and R-chart upper control limit (UCL) and lower control limit (LCL) assuming no standards given.

Solution

The overall ~~range~~ is

Replace with: "average"

$$\overline{X}_W = \sum_{i=1}^{K} \frac{\overline{X}_i}{K}$$

$$= \frac{\begin{array}{c} 20.00 + 20.00 + 20.00 + 20.33 + 20.00 \\ +20.33 + 20.00 + 20.33 + 20.33 + 20.33 \end{array}}{10}$$

Example 62.19

A factory dispenses candy into individual bags. Three observations of the process are made every 2 hr until 10 overall samples have been taken. The results of those observations are as shown.

sample	observations			$\overline{X}_i$	r_i
1	20	19	21	20.00	2
2	20	20	20	20.00	0
3	21	20	19	20.00	2
4	19	22	20	20.33	3
5	21	20	19	20.00	2
6	20	20	21	20.33	1
7	19	20	21	20.00	2
8	20	22	19	20.33	3
9	19	20	22	20.33	3
10	20	21	20	20.33	1

Determine the X-chart and R-chart upper control limit (UCL) and lower control limit (LCL) assuming no standards given.

Solution

The overall ~~range~~ is

Replace with: "average"

Statistical Quality Control

$$\overline{X}_W = \sum_{i=1}^{K} \frac{\overline{X}_i}{K} = \frac{\begin{array}{c}20.00 + 20.00 + 20.00 + 20.33 + 20.00\\ +20.33 + 20.00 + 20.33 + 20.33 + 20.33\end{array}}{10} = 20.17$$

The average range is

Statistical Quality Control

$$\overline{R} = \sum_{i=1}^{K} \frac{r_i}{K} = \frac{\begin{array}{c}2+0+2+3+2\\ +1+2+3+3+1\end{array}}{10} = 1.90$$

For three observations, the control-chart limit factors are $A_2 = 1.02$, $D_3 = 0$, and $D_4 = 2.57$. [Factors for Control-Chart Limits]

The X-chart UCL is

Statistical Quality Control

$$\text{UCL}_{\overline{X}_W,\text{NO STD}} = \overline{X}_W + A_2\overline{R} = 20.17 + (1.02)(1.90) = 22.11$$

The X-chart LCL is

$$\boxed{20.17 - (1.02)(1.90)}$$

$$\mathrm{LCL}_{\overline{X}_W,\mathrm{NO\ STD}} = \overline{X}_W - A_2\overline{R} = \cancel{20.17 + (1.02)\,(1.90)} = 18.23$$

The R-chart UCL is

$$\mathrm{UCL}_{\overline{R}\ \mathrm{NO\ STD}} = D_4\overline{R} = (2.57)\,(1.90) = 4.88$$

The R-chart LCL is

$$\mathrm{LCL}_{\overline{R},\mathrm{NO\ STD}} = D_3\overline{R} = (0)\,(1.90) = 0$$

Since no observations are out of the observation and range control limits, the process can be considered stable.

With typical bridge trusses supported at the ends and loaded downward at the joints, the upper chords are almost always in compression, and the end panels and lower chords are almost always in tension.

32. DETERMINATE TRUSSES

A truss will be statically determinate if Eq. 44.53 holds.

$$\text{no. of members} = 2(\text{no. of joints}) - 3 \qquad 44.53$$

If the left-hand side is greater than the right-hand side (i.e., there are *redundant members*), the truss is statically indeterminate. If the left-hand side is less than the right-hand side, the truss is unstable and will collapse under certain types of loading.

Equation 44.53 is a special case of the following general criterion.

$$\begin{aligned} &\text{no. of members} \\ &\quad + \text{no. of reactions} \\ &\qquad - 2(\text{no. of joints}) = 0 \quad [\text{determinate}] \\ &\qquad\qquad\qquad\qquad\quad > 0 \quad [\text{indeterminate}] \\ &\qquad\qquad\qquad\qquad\quad < 0 \quad [\text{unstable}] \end{aligned} \qquad 44.54$$

Furthermore, Eq. 44.53 is a necessary, but not sufficient, condition for truss stability. It is possible to arrange the members in such a manner as to not contribute to truss stability. This will seldom be the case in actual practice, however.

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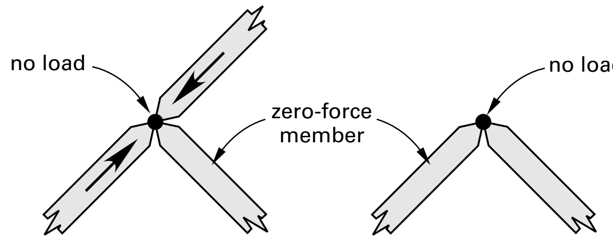
Replace with the text from the following page.

33. ZERO-FORCE MEMBERS

~~Forces in truss members can sometimes be determined by inspection. One of these cases is where there are *zero-force members*. A third member framing into a joint already connecting two collinear members carries no internal force unless there is a load applied at that joint. Similarly, both members forming an apex of the truss are zero-force members unless there is a load applied at the apex.~~ (See Fig. 44.15.)

Figure 44.15 *Zero-Force Members*



34. METHOD OF JOINTS

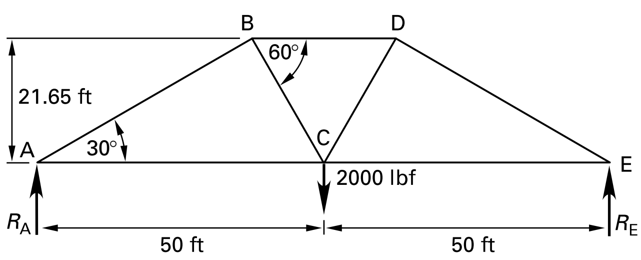
The *method of joints* is one of three methods that can be used to find the internal forces in each truss member. This method is useful when most or all of the truss member forces are to be calculated. Because this method advances from joint to adjacent joint, it is inconvenient when a single isolated member force is to be calculated.

The method of joints is a direct application of the equations of equilibrium in the x- and y-directions. Traditionally, the method starts by finding the reactions supporting the truss. Next, the joint at one of the reactions is evaluated, which determines all the member forces framing into the joint. Then, knowing one or more of the member forces from the previous step, an adjacent joint is analyzed. The process is repeated until all the unknown quantities are determined.

At a joint, there may be up to two unknown member forces, each of which can have dependent x- and y-components.[14] Since there are two equilibrium equations, the two unknown forces can be determined. Even though determinate, however, the sense of a force will often be unknown. If the sense cannot be determined by logic, an arbitrary decision can be made. If the incorrect direction is chosen, the calculated force will be negative.

Example 44.7

Use the method of joints to calculate the force **BD** in the truss shown.



Solution

First, find the reactions. Assume clockwise is positive and take moments about point A.

$$\begin{aligned} \sum M_A &= (2000\ \text{lbf})(50\ \text{ft}) - R_E(50\ \text{ft} + 50\ \text{ft}) = 0 \\ R_E &= 1000\ \text{lbf} \end{aligned}$$

Since the sum of forces in the y-direction is also zero,

$$\begin{aligned} \sum F_y &= R_A + 1000\ \text{lbf} - 2000\ \text{lbf} = 0 \\ R_A &= 1000\ \text{lbf} \end{aligned}$$

There are three unknowns at joint B (and also at D). The analysis must start at joint A (or E) where there are only two unknowns (forces **AB** and **AC**).

[14] Occasionally, there will be three unknown member forces. In that case, an additional equation must be derived from an adjacent joint.

Statics

Zero-Force Members

Forces in truss members can sometimes be determined by inspection. One of these cases is where there are zero-force member. Zero-force members can be determined by evaluating if the other members joined to the member in question through the pinned connection can react to an internal load in the member. If not, the load in the member must be zero. The first typical configuration for a zero-force member is when a third member framing into a joint already connecting two collinear members must carry no internal force unless there is a load applied at that joint. The second typical configuration is two members forming an apex of a truss with no load applied to the apex.

Gases, of course, are easily compressed. The compressibility of an ideal gas depends on its pressure, p, its ratio of specific heats, k, and the nature of the process.[26] Depending on the process, the compressibility may be known as *isothermal compressibility* or (*adiabatic*) *isentropic compressibility*. Of course, compressibility is zero for constant-volume processes and is infinite (or undefined) for constant-pressure processes.

$$\beta_T = \frac{1}{p} \quad \text{[isothermal ideal gas processes]} \qquad 14.37$$

$$\beta_s = \frac{1}{kp} \quad \text{[adiabatic ideal gas processes]} \qquad 14.38$$

Table 14.10 *Approximate Compressibilities of Common Liquids at 1 atm*

liquid	temperature	β (in^2/lbf)	β (1/atm)
mercury	32°F	0.027×10^{-5}	0.39×10^{-5}
glycerin	60°F	0.16×10^{-5}	2.4×10^{-5}
water	60°F	0.33×10^{-5}	4.9×10^{-5}
ethyl alcohol	32°F	0.68×10^{-5}	10×10^{-5}
chloroform	32°F	0.68×10^{-5}	10×10^{-5}
gasoline	60°F	1.0×10^{-5}	15×10^{-5}
hydrogen	20K	11×10^{-5}	160×10^{-5}
helium	2.1K	48×10^{-5}	700×10^{-5}

(Multiply 1/psi by 14.696 to obtain 1/atm.)

(Multiply in^2/lbf by 0.145 to obtain 1/kPa.)

Replace with:
"2.2 x 10^6 kPa"

Example 14.7

Water at 68°F (20°C) and 1 atm has a density of 62.3 lbm/ft^3 (997 kg/m^3). The bulk modulus has a constant value of 320,000 lbf/in^2 (~~2.2 × 106 kPa~~). What is the new density if the pressure is isothermally increased from 14.7 lbf/in^2 to 400 lbf/in^2 (100 kPa to 2760 kPa)?

SI Solution

Compressibility is the reciprocal of the bulk modulus. From Eq. 14.35,

$$\beta = \frac{1}{B} = \frac{1}{2.2 \times 10^6 \text{ kPa}} = 4.55 \times 10^{-7} \text{ 1/kPa}$$

From Eq. 14.36,

$$\begin{aligned}
\rho_2 &= \rho_1\big(1 + \beta(p_2 - p_1)\big) \\
&= \left(997 \ \frac{\text{kg}}{\text{m}^3}\right)\left(\begin{aligned}&1 + \left(4.55 \times 10^{-7} \ \frac{1}{\text{kPa}}\right) \\ &\quad \times (2760 \text{ kPa} - 100 \text{ kPa})\end{aligned}\right) \\
&= 998.2 \text{ kg/m}^3
\end{aligned}$$

Customary U.S. Solution

Compressibility is the reciprocal of the bulk modulus. From Eq. 14.35,

$$\beta = \frac{1}{B} = \frac{1}{320{,}000 \ \frac{\text{lbf}}{\text{in}^2}} = 0.3125 \times 10^{-5} \text{ in}^2/\text{lbf}$$

From Eq. 14.36,

$$\begin{aligned}
\rho_2 &= \rho_1\big(1 + \beta(p_2 - p_1)\big) \\
&= \left(62.3 \ \frac{\text{lbm}}{\text{ft}^3}\right)\left(\begin{aligned}&1 + \left(0.3125 \times 10^{-5} \ \frac{\text{in}^2}{\text{lbf}}\right) \\ &\quad \times \left(400 \ \frac{\text{lbf}}{\text{in}^2} - 14.7 \ \frac{\text{lbf}}{\text{in}^2}\right)\end{aligned}\right) \\
&= 62.38 \text{ lbm/ft}^3
\end{aligned}$$

18. BULK MODULUS

The *bulk modulus*, B, of a fluid is analogous to the modulus of elasticity of a solid. Typical units are lbf/in^2, atm, and kPa. The term dp in Eq. 14.39 represents an increase in stress. The term dV/V_0 is a *volumetric strain*. Analogous to Hooke's law describing elastic formation, the *bulk modulus* of a fluid (liquid or gas) is given by Eq. 14.39.

Stress, Pressure, and Viscosity

$$B = \frac{\text{stress}}{\text{strain}} = -\frac{dp}{\frac{dV}{V}} \qquad 14.39$$

The bulk modulus can also be written in terms of partial derivatives.

$$B = -V\left(\frac{\partial p}{\partial V}\right)_T \qquad 14.40$$

The term *secant bulk modulus* is associated with Eq. 14.39 (the average slope), while the terms *tangent bulk modulus* and *point bulk modulus* are associated with Eq. 14.40 (the instantaneous slope).

[26] For air, $k = 1.4$.

6. IMPLICIT DIFFERENTIATION

When a relationship between n variables cannot be manipulated to yield an explicit function of $n-1$ independent variables, that relationship implicitly defines the nth variable. Finding the derivative of the implicit variable with respect to any other independent variable is known as *implicit differentiation*.

An implicit derivative is the quotient of two partial derivatives. The two partial derivatives are chosen so that dividing one by the other eliminates a common differential. For example, if z cannot be explicitly extracted from $f(x,y,z)=0$, the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$ can still be found as follows.

$$\frac{\partial z}{\partial x} = \frac{\dfrac{\partial f}{\partial x}}{\dfrac{\partial f}{\partial z}} \qquad 8.34$$

$$\frac{\partial z}{\partial y} = \frac{\dfrac{\partial f}{\partial y}}{\dfrac{\partial f}{\partial z}} \qquad 8.35$$

Example 8.7

Find the derivative dy/dx of

$$f(x,y) = x^2 + xy + y^3$$

Solution

Implicit differentiation is required because x cannot be extracted from $f(x,y)$.

$$\frac{\partial f}{\partial x} = 2x + y$$

$$\frac{\partial f}{\partial y} = x + 3y^2$$

$$\frac{dy}{dx} = \frac{-\dfrac{\partial f}{\partial x}}{\dfrac{\partial f}{\partial y}} = \frac{-(2x+y)}{x+3y^2}$$

Example 8.8

Solve Ex. 8.6 using implicit differentiation.

Solution

$$f(x,y,z) = x^2 + y^2 + z^2 - 9 = 0$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial z} = 2z$$

$$\frac{\partial z}{\partial x} = \frac{-\dfrac{\partial f}{\partial x}}{\dfrac{\partial f}{\partial z}} = \frac{-2x}{2z} = -\frac{x}{z}$$

At the point $(1,2,2)$,

$$\frac{\partial z}{\partial x} = -\frac{1}{2}$$

7. TANGENT PLANE FUNCTION

Partial derivatives can be used to find the equation of a plane tangent to a three-dimensional surface defined by $f(x,y,z)=0$ at some point, P_0.

$$\begin{aligned} T(x_0,y_0,z_0) &= (x-x_0)\left.\frac{\partial f(x,y,z)}{\partial x}\right|_{\mathrm{P}_0} \\ &\quad + (y-y_0)\left.\frac{\partial f(x,y,z)}{\partial y}\right|_{\mathrm{P}_0} \\ &\quad + (z-z_0)\left.\frac{\partial f(x,y,z)}{\partial z}\right|_{\mathrm{P}_0} \\ &= 0 \end{aligned} \qquad 8.36$$

The coefficients of x, y, and z are the same as the coefficients of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ of the normal vector at point P_0. (See Sec. 8.10.)

Example 8.9

What is the equation of the plane that is tangent to the surface defined by $f(x,y,z) = 4x^2 + y^2 - 16z = 0$ at the point $(2,4,2)$?

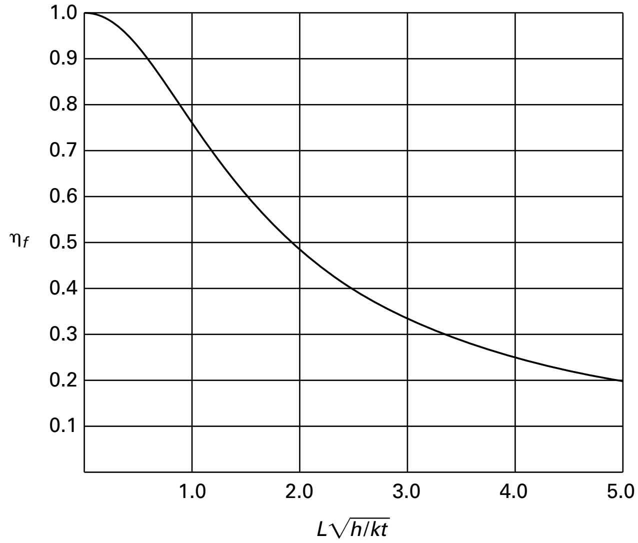
Solution

Calculate the partial derivatives and substitute the coordinates of the point.

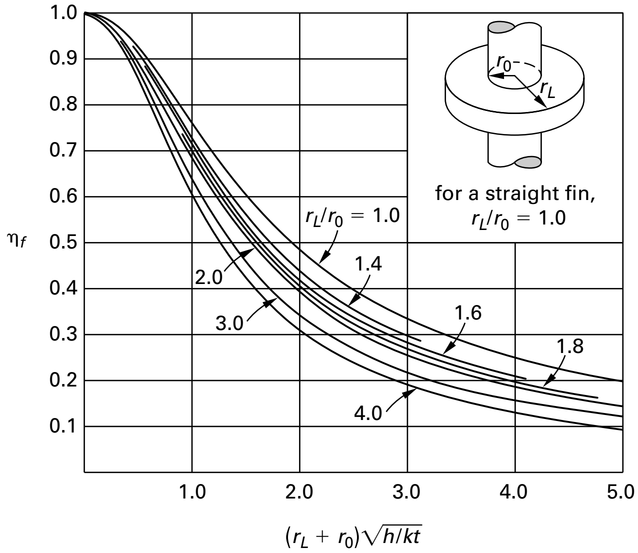
$$\left.\frac{\partial f(x,y,z)}{\partial x}\right|_{\mathrm{P}_0} = 8x\Big|_{(2,4,2)} = (8)(2) = 16$$

$$\left.\frac{\partial f(x,y,z)}{\partial y}\right|_{\mathrm{P}_0} = 2y\Big|_{(2,4,2)} = (2)(4) = 8$$

Figure 33.5 *Fin Efficiency (refer to Fig. 33.6 for dimensions of fins)*



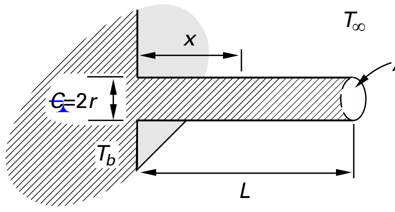
(a) rectangular (straight) fins



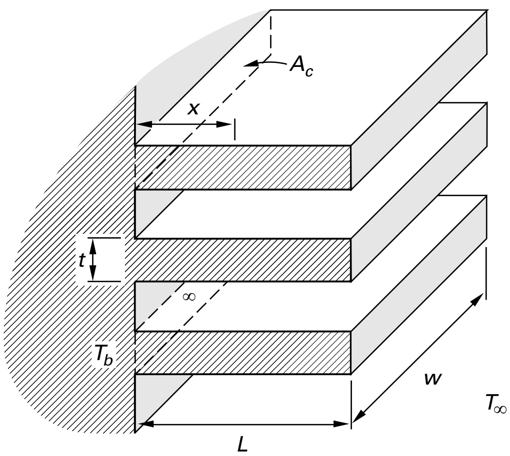
(b) circular (transverse) fins

Adapted from *Engineering Heat Transfer*, by James R. Welty, John Wiley & Sons, copyright © 1974.

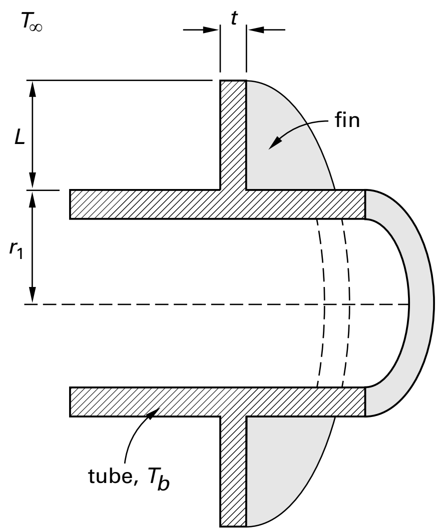
Figure 33.6 *Finned Heat Radiators*



(a) finite cylindrical fin (pin fin)



(b) finite rectangular fin (straight fin)



(c) annular (circular) fin (transverse fin)

coefficient for the fin can be used as an approximation.[34] Closed form solutions for most common nonadiabatic cases are available in heat transfer textbooks.

Fins should not be used inside fluid-carrying pipes. Aside from the difficulty of manufacturing, spiral or transverse fins within a pipe would trap fluid and substantially decrease the internal film coefficient. Longitudinal fins would "laminarize" the flow, also decreasing the internal film coefficient.

[34]Even this is an approximation, as the additional heat transfer will change the temperature distribution along the fin length. However, the effect is probably minimal.

18. AIR WASHERS

An *air washer* is a device that passes air through a dense spray of recirculating water. The water is used to change the properties of the air. Air washers are used in air purifying and cleaning processes (i.e., removal of solids, liquids, gases, vapors, and odors), as well as for evaporative cooling and dew-point control.[5]

The difference between a spray humidifier and spray dehumidifier is the temperature of the spray water. In an *adiabatic air washer*, the spray water is recirculated without being heated or cooled. After equilibrium is reached, the water temperature will be equal to the air's entering wet-bulb temperature. The air will be cooled and humidified, leaving partially or completely saturated at its entering wet-bulb temperature. However, if the spray water is chilled, the air will be cooled and dehumidified. And, if the spray water is heated, the air will be humidified and (possibly) heated.

An air washer's *saturation efficiency*, typically 90% to 95%, is measured by the drop in dry-bulb temperature relative to the entering wet-bulb depression.

$$\eta_{\text{sat}} = \frac{T_{\text{in,db}} - T_{\text{out,db}}}{T_{\text{in,db}} - T_{\text{in,wb}}} \quad 37.48$$

Air velocity through washers is approximately 500 ft/min (2.6 m/s). Velocities outside the range of 300 ft/min to 750 ft/min (1.5 m/s to 3.8 m/s) are probably faulty. The water pressure is typically 20 psig to 40 psig (140 kPa to 280 kPa). The spray quantity per bank of nozzles is in the range of 1.5 gal/min to 5 gal/min per 1000 ft^3 (3.3 L/s to 11 L/s per 1000 m^3) of air. Screens, louvers, and mist eliminator plates will generate a static pressure drop of approximately 0.2 in wg to 0.5 in wg (50 kPa to 125 kPa) at 500 ft/min (2.6 m/s). Other operating parameters used to describe air washer performance include air mass flow rate per unit area (lbm/hr-ft^2 or kg/m^2·s), air and liquid heat transfer coefficients per volume of chamber (Btu/hr-°F-ft^3 or kW/°C·m^3), and the *spray ratio* (the mass of water sprayed to the mass of air passing through the washer per unit time).

HVAC

19. COOLING WITH HUMIDIFICATION

When air passes through a water spray (as in an *air washer*), an *adiabatic saturation process* known as *evaporative cooling* occurs.[6] (See Sec. 37.17.) The air leaves with a lower temperature and a higher moisture content. This is graphically represented on the psychrometric chart by a condition line parallel to the lines of constant enthalpy (essentially constant wet-bulb temperature), and is mathematically represented using the ratio of the change in enthalpy to the change in the humidity ratio.

Adiabatic Mixing of Water Injected Into Moist Air (Evaporative Cooling)

$$\frac{h_2 - h_1}{W_2 - W_1} = \frac{\Delta h}{\Delta W} = h_w \quad 37.49$$

Adiabatic saturation is a constant-enthalpy process, since any evaporation of the water requires heat to be drawn from the air. Since the removed heat goes into the remaining water, the water temperature increases. When the water spray is continuously recirculated, the water temperature gradually increases to the wet-bulb temperature of the incoming air. The minimum leaving air temperature will be the water temperature (i.e., the wet-bulb temperature of the incoming air).

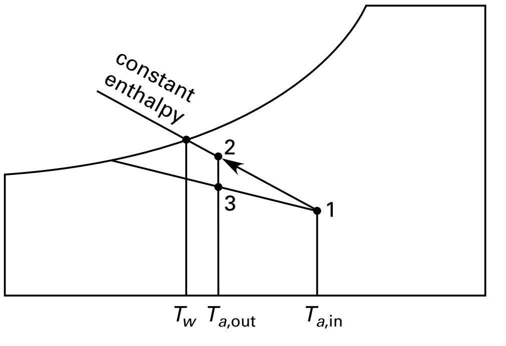
The direct saturation efficiency, η_e, is the extent to which the temperature of the air leaving an evaporative cooler approaches the wet-bulb temperature of the air entering the cooler.

Direct Evaporative Air Coolers

$$\epsilon_e = 100\frac{T_1 - T_2}{T_1 - T_{\text{wb,in}}} \quad 37.50$$

During steady-state operation, the temperature of the water spray will normally be stable at the air's wet-bulb temperature. However, the water temperature can also be artificially maintained by refrigeration at less than the wet-bulb temperature (but more than the dew-point temperature). Line 1–3 in Fig. 37.5 illustrates such a process.

Figure 37.5 Cooling with Humidification (adiabatic saturation)



To prevent ice buildup, the cooled air temperature should be kept from dropping below the freezing point of water. The entering wet-bulb temperature should be kept above 35°F (1.7°C).

[5]Air washers are generally not used for removing carbonaceous or greasy particles.

[6]An *air washer* is basically a *spray chamber* through which air passes. When supplied with chilled water from a refrigeration source, the air washer can cool, dehumidify, or humidify the air. Air washers can be used without refrigeration to cool and humidify the air through an evaporative cooling process.

the three coordinate axes, x, y, and z, respectively. (There are other methods of representing vectors, in addition to bold letters. For example, the unit vector **i** is represented as $\bar{i}$ or $\hat{i}$ in other sources.) Unit vectors are used in vector equations to indicate direction without affecting magnitude. For example, the vector representation of a 97 N force in the negative x-direction would be written as $\mathbf{F} = -97\mathbf{i}$.

4. CONCENTRATED FORCES

A *force* is a push or pull that one body exerts on another. A *concentrated force*, also known as a *point force*, is a vector having magnitude, direction, and location (i.e., point of application) in three-dimensional space. (See Fig. 44.1.) In this chapter, the symbols **F** and F will be used to represent the vector and its magnitude, respectively. (As with the unit vectors, the symbols $\mathbf{F}$, $\bar{F}$, and $\hat{F}$ are used in other sources to represent the same vector.)

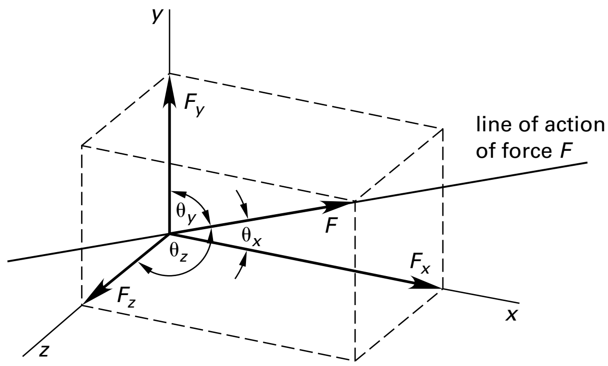
The vector representation of a three-dimensional force is given by Eq. 44.1, and the vector representation of a plane force is given by Eq. 44.2. Of course, vector addition is required.

Force

$$F = F_x i + F_y j \tag{44.1}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \tag{44.2}$$

Figure 44.1 *Components and Direction Angles of a Force*



If **u** is a *unit vector* in the direction of the force, the force can be represented as

$$\mathbf{F} = F\mathbf{u} \tag{44.3}$$

The components of the force can be found from the *direction cosines*, the cosines of the true angles made by the force vector with the x-, y-, and z-axes.

Resolution of a Force

$$F_x = F\cos\theta_x \tag{44.4}$$

$$F_y = F\cos\theta_y \tag{44.5}$$

$$F_z = F\cos\theta_z \tag{44.6}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \tag{44.7}$$

The *line of action* of a force is the line in the direction of the force extended forward and backward. The force, **F**, and its unit vector, **u**, are along the line of action.

5. MOMENTS

Moment is the name given to the tendency of a force to rotate, turn, or twist a rigid body about an actual or assumed pivot point. (Another name for moment is *torque*, although torque is used mainly with shafts and other power-transmitting machines.) When acted upon by a moment, unrestrained bodies rotate. However, rotation is not required for the moment to exist. When a restrained body is acted upon by a moment, there is no rotation.

An object experiences a moment whenever a force is applied to it.[3] Only when the line of action of the force passes through the center of rotation (i.e., the actual or assumed pivot point) will the moment be zero.

Moments have primary dimensions of length × force. Typical units include foot-pounds, inch-pounds, and newton-meters.[4]

6. MOMENT OF A FORCE ABOUT A POINT

Moments are vectors. The moment vector, $\mathbf{M}_\text{O}$, for a force about point O is the *cross product* of the force, **F**, and the vector from point O to the point of application of the force, known as the *position vector*, **r**. The scalar product $|\mathbf{r}|\sin\theta$ is known as the *moment arm*, d.

Moments (Couples)

$$\mathbf{M}_\text{O} = \mathbf{r} \times \mathbf{F} \tag{44.8}$$

$$M_\text{O} = |\mathbf{M}_\text{O}| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta = d|\mathbf{F}| \quad [\theta \le 180°] \tag{44.9}$$

The line of action of the moment vector is normal to the plane containing the force vector and the position vector. The sense (i.e., the direction) of the moment is determined from the *right-hand rule*.

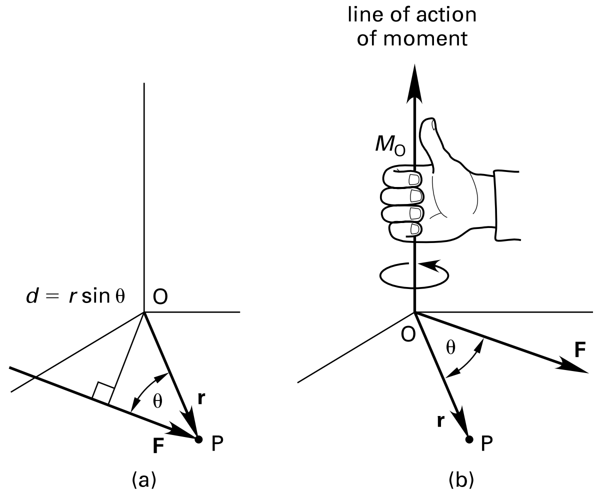
Right-hand rule: Place the position and force vectors tail to tail. Close your right hand and position it over the pivot point. Rotate the position vector into the force

[3]The moment may be zero, as when the moment arm length is zero, but there is a (trivial) moment nevertheless.

[4]Units of kilogram-force-meter have also been used in metricated countries. Foot-pounds and newton-meters are also the units of energy. To distinguish between moment and energy, some authors reverse the order of the units, so pound-feet and meter-newtons become the units of moment. This convention is not universal and is unnecessary since the context is adequate to distinguish between the two.

vector, and position your hand such that your fingers curl in the same direction as the position vector rotates. Your extended thumb will coincide with the direction of the moment.[5] (See Fig. 44.2.)

Figure 44.2 *Right-Hand Rule*



7. VARIGNON'S THEOREM

Varignon's theorem is a statement of how the total moment is derived from a number of forces acting simultaneously at a point. It states that the sum of individual moments about a point caused by multiple concurrent forces is equal to the moment of the resultant force about the same point.

$$\cancel{(\mathbf{r}\times\mathbf{F}_1)+(\mathbf{r}\times\mathbf{F}_2)+\cdots=\mathbf{r}\times(\mathbf{F}_1+\mathbf{F}_2+\cdots)} \qquad \text{~~44.10~~}$$

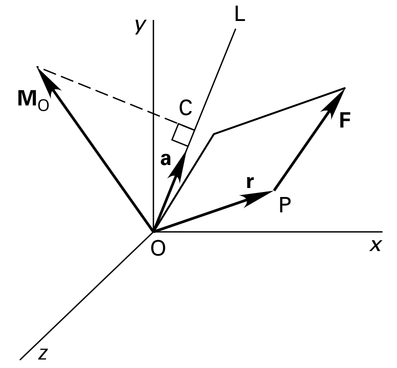
~~Systems of n Forces~~

$$\cancel{\mathbf{M}=\sum M_n=\sum r_n\times F_n} \qquad \text{~~44.11~~}$$

8. MOMENT OF A FORCE ABOUT A LINE

Most rotating machines (motors, pumps, flywheels, etc.) have a fixed rotational axis. That is, the machines turn around a line, not around a point. The moment of a force about the rotational axis is not the same as the moment of the force about a point. In particular, the moment about a line is a scalar.[6] (See Fig. 44.3.)

Figure 44.3 *Moment of a Force About a Line*



Moment M_{OL} of force $\mathbf{F}$ about line OL is the projection, OC, of moment $\mathbf{M}_\mathrm{O}$ onto the line. Equation 44.12 gives the moment of a force about a line. $\mathbf{a}$ is the unit vector directed along the line, and a_x, a_y, and a_z are the direction cosines of the axis OL. Notice that Eq. 44.12 is a dot product (i.e., a scalar).

$$\begin{aligned} M_{\mathrm{OL}} &= \mathbf{a}\cdot\mathbf{M}_\mathrm{O} = \mathbf{a}\cdot(\mathbf{r}\times\mathbf{F}) \\ &= \begin{vmatrix} a_x & a_y & a_z \\ x_\mathrm{P}-x_\mathrm{O} & y_\mathrm{P}-y_\mathrm{O} & z_\mathrm{P}-z_\mathrm{O} \\ F_x & F_y & F_z \end{vmatrix} \end{aligned} \qquad 44.12$$

If point O is the origin, then Eq. 44.12 will reduce to Eq. 44.13.

$$M_{\mathrm{OL}} = \begin{vmatrix} a_x & a_y & a_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \qquad 44.13$$

9. COMPONENTS OF A MOMENT

The direction cosines of a force (vector) can be used to determine the components of the moment about the coordinate axes.

$$M_x = M\cos\theta_x \qquad 44.14$$

$$M_y = M\cos\theta_y \qquad 44.15$$

$$M_z = M\cos\theta_z \qquad 44.16$$

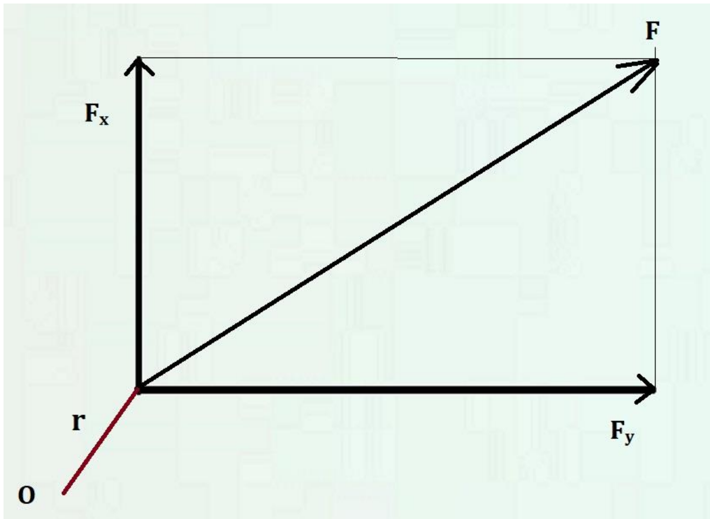
Alternatively, the following three equations can be used to determine the components of the moment from a force applied at point (x, y, z) referenced to an origin at $(0, 0, 0)$.

[5]The direction of a moment also corresponds to the direction a right-hand screw would progress if it was turned in the direction that rotates $\mathbf{r}$ into $\mathbf{F}$.
[6]Some sources say that the moment of a force about a line can be interpreted as a moment directed along the line. However, this interpretation does not follow from vector operations.

Insert A

Consider forces $\mathbf{F_x}$ and $\mathbf{F_y}$ acting simultaneously (see Fig. 44.3). The resultant force is **F**. Varignon's theorem states that the moment of **F** about a given point of reference, O, is equal to the sum of the moments of $\mathbf{F_x}$ and $\mathbf{F_y}$ about the same reference point. In Fig. 44.3, **r** is the position vector from reference point O to the point of action of force **F**.

Figure 44.3 Varignon's Theorem



Take moments about the reference point O. Each moment is the cross product of the position vector and the force. According to Varignon's theorem,

$$\begin{aligned}\mathbf{M}_{\mathrm{O}} &= \mathbf{r}\times\mathbf{F}_{\mathrm{x}} + \mathbf{r}\times\mathbf{F}_{\mathrm{y}} \\ &= \mathbf{r}\times\left(\mathbf{F}_{\mathrm{x}}+\mathbf{F}_{\mathrm{y}}\right) \\ &= \mathbf{r}\times\mathbf{F}\end{aligned} \qquad 44.10$$

Varignon's theorem can also be applied to a system of coplanar forces, $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$, ..., $\mathbf{F_n}$. If **F** is the resultant of the system of forces, then, by Varignon's theorem,

Systems of n Forces

$$\begin{aligned}\mathbf{M} &= \sum\mathbf{M}_{\mathrm{n}} = \sum\left(\mathbf{r}_{\mathrm{n}}\times\mathbf{F}_{\mathrm{n}}\right) \\ &= \mathbf{r}\times\mathbf{F}_1 + \mathbf{r}\times\mathbf{F}_2 + \mathbf{r}\times\mathbf{F}_3 + \cdots + \mathbf{r}\times\mathbf{F}_{\mathrm{n}} \\ &= \mathbf{r}\times\left(\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3+\cdots+\mathbf{F}_{\mathrm{n}}\right) \\ &= \mathbf{r}\times\mathbf{F}\end{aligned} \qquad 44.11$$

Figure 47.12 *Crack Length and Geometrical Factors*

When a is measured in meters and σ is measured in MPa, typical units for both the stress intensity factor and fracture toughness are MPa $\sqrt{\text{m}}$. Typical values of fracture toughness for various materials are given in Table TBD4.

Table 47.5 *Table TBD4 Representative Values of Fracture Toughness*

material	K_{LC} MPa $\sqrt{\text{m}}$	K_{LC} ~~MPa $\sqrt{\text{m}}$~~
Al 2014-T651	24.2	22
Al 2024-T3	44	40
52100 steel	14.3	13
4340 steel	46	42
alumina	4.5	4.1
silicon carbide	3.5	3.2

Example 47.2

An aluminum alloy plate containing a 2 cm long crack is 10 cm wide and 0.5 cm thick. The plate is pulled with a uniform tensile force of 10000 N. What is most nearly the stress intensity factor at the end of the crack?

(A) 2.1 MPa $\sqrt{\text{m}}$

(B) 5.5 MPa $\sqrt{\text{m}}$

(C) 12 MPa $\sqrt{\text{m}}$

(D) 21 MPa $\sqrt{\text{m}}$

Solution

From Eq. 48.1, the nominal engineering stress is

Uniaxial Loading and Deformation

$$\sigma = \frac{P}{A_o} = \frac{\left(10\,000\ \text{N}\right)\left(100\ \frac{\text{cm}}{\text{m}}\right)^2}{(10\ \text{cm})(0.5\ \text{cm})} \qquad 47.16$$

$$= 20 \times 10^6\ \frac{\text{N}}{\text{m}^2} \quad (20\ \text{MPa})$$

Since this is an exterior crack, Y=1.1. Using Eq. TBD2, the stress intensity factor is

Mechanical Properties

$$K_{\text{LC}} = Y\sigma\sqrt{\pi a}$$

$$= (1.1)(20\ \text{MPa})\sqrt{\pi\left(\frac{2\ \text{cm}}{100\ \frac{\text{cm}}{\text{m}}}\right)} \qquad 47.17$$

$$= 5.51\ \text{MPa}\cdot\sqrt{\text{m}} \quad (5.5\ \text{MPa}\cdot\sqrt{\text{m}})$$

The answer is (B).

11. STRAIN ENERGY

Strain energy, also known as *internal work*, is the energy per unit volume stored in a deformed material. The strain energy is equivalent to the work done by the applied tensile force. Simple work is calculated as the product of a force moving through a distance.

$$\text{work} = \text{force} \times \text{distance} = \int F\ dL \qquad 47.18$$

$$\text{work per unit volume} = \int \frac{F\ dL}{AL} = \int_0^{\epsilon_{\text{final}}} \sigma d\epsilon \qquad 47.19$$

This work per unit volume corresponds to the area under the true stress-strain curve. Units are in-lbf/in^3 (i.e., inch-pounds (a unit of energy) per cubic inch (a unit of volume)), usually shortened to lbf/in^2, or J/m^3 (i.e., joules per cubic meter), less frequently shown as Pa. (Equation 47.19 cannot be simplified further because stress is not proportional to strain for the entire curve; however, for the elastic region, the stress-strain curve is essentially a straight line, and the area is triangular Therefore, if a body of length L is deformed under force F or torque T, the resulting strain energy be expressed in terms of Eq. TBD through Eq. TBD4.

Strain Energy

$$U = \tfrac{1}{2}F\delta \quad \left[\text{strain energy}\right] \qquad 47.20$$

$$U = \frac{F^2L}{2AE} \quad \left[\text{tension or compression}\right] \qquad 47.21$$

$$U = \frac{T^2L}{2GJ} \quad \left[\text{torsion}\right] \qquad 47.22$$

$$U = \frac{F^2L}{2AG} \quad \left[\text{shear}\right] \qquad 47.23$$

$$U = \int \frac{M^2\ dx}{2EI} \quad \left[\text{bending}\right] \qquad 47.24$$

(b) t, ω_0, and ω are known, and θ is unknown.

Normal and Tangential Components

$$\theta(t)_0 = \alpha_0 \frac{(t-t_0)^2}{2} + \omega_0(t-t_0) + \theta$$

$$= \left(-5.236\,\frac{\text{rad}}{\text{sec}^2}\right)\left(\frac{(8\text{ sec} - 0\text{ sec})^2}{2}\right) + \left(41.89\,\frac{\text{rad}}{\text{sec}}\right)(8\text{ sec} - 0\text{ sec}) + 0\text{ rad}$$

$$= 167.6\text{ rad} \quad (26.67\text{ rev})$$

12. RELATIONSHIP BETWEEN LINEAR AND ROTATIONAL VARIABLES

A particle moving in a curvilinear path will also have instantaneous linear velocity and linear acceleration. These linear variables will be directed tangentially to the path and, therefore, are known as *tangential velocity* and *tangential acceleration*, respectively. (See Fig. 56.6.) In general, the linear variables can be obtained by multiplying the rotational variables by the path radius, r.

Plane Circular Motion

$$\mathrm{v}_t = r\omega \qquad 56.19$$

$$\mathrm{v}_{t,x} = \mathrm{v}_t \cos\phi = \omega r \cos\phi \qquad 56.20$$

$$\mathrm{v}_{t,y} = \mathrm{v}_t \sin\phi = \omega r \sin\phi \qquad 56.21$$

Plane Circular Motion

$$a_t = r\alpha \qquad 56.22$$

If the path radius is constant, as it would be in rotational motion, the linear distance (i.e., the *arc length*) traveled is

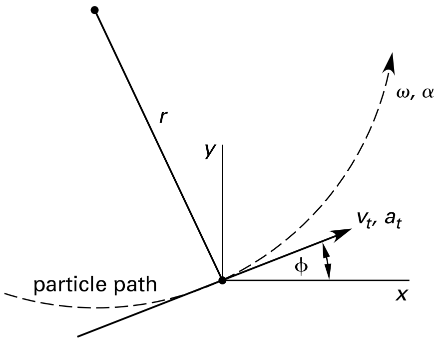
Plane Circular Motion

$$s = r\theta \qquad 56.23$$

13. NORMAL ACCELERATION

A moving particle will continue tangentially to its path unless constrained otherwise. For example, a rock twirled on a string will move in a circular path only as long as there is tension in the string. When the string is released, the rock will move off tangentially.

Figure 56.6 Tangential Variables



The twirled rock is acted upon by the tension in the string. In general, a restraining force will be directed toward the center of rotation. Whenever a mass experiences a force, an acceleration is acting.[4] The acceleration has the same sense as the applied force (i.e., is directed toward the center of rotation). Since the inward acceleration is perpendicular to the tangential velocity and acceleration, it is known as *normal acceleration*, a_n. (See Fig. 56.7.)

Plane Circular Motion

$$a_n = -r\omega^2 \text{Eq. TBD}$$

$$a_n = \frac{\mathrm{v}_t^2}{r} = \mathrm{v}_t\omega \qquad 56.24$$

The *resultant acceleration*, a, is the vector sum of the tangential and normal accelerations. The magnitude of the resultant acceleration is

$$a = \sqrt{a_t^2 + a_n^2} \qquad 56.25$$

The x- and y-components of the resultant acceleration are

$$a_x = a_n \sin\theta \pm a_t \cos\theta \qquad 56.26$$

$$a_y = a_n \cos\theta \mp a_t \sin\theta \qquad 56.27$$

The normal and tangential accelerations can be expressed in terms of the x- and y-components of the resultant acceleration (not shown in Fig. 56.7).

$$a_n = a_x \sin\theta \pm a_y \cos\theta \qquad 56.28$$

$$a_t = a_x \cos\theta \mp a_y \sin\theta \qquad 56.29$$

[4]This is a direct result of Newton's second law of motion.

For ideal gases, the propagation velocity is calculated from Eq. 72.13.

$$c = \sqrt{kRT} = \sqrt{\frac{kR^*T}{M}} \quad \text{[SI]} \qquad 72.13(a)$$

$$c = \sqrt{kg_cRT} = \sqrt{\frac{kg_cR^*T}{M}} \quad \text{[U.S.]} \qquad 72.13(b)$$

Table 72.3 lists approximate values for the speed of sound in various media.

Table 72.3 *Approximate Speeds of Sound (at one atmospheric pressure)*

	speed of sound	
material	(ft/sec)	(m/s)
air	1130 at 70°F	330 at 0°C
aluminum	16,400	4990
carbon dioxide	870 at 70°F	260 at 0°C
hydrogen	3310 at 70°F	1260 at 0°C
steel	16,900	5150
water	4880 at 70°F	1490 at 20°C

(Multiply ft/sec by 0.3048 to obtain m/s.)

11. DOPPLER EFFECT

If the distance between a sound source and a listener is changing, the frequency heard will differ from the frequency emitted. If the separation distance is decreasing, the frequency will be shifted higher; if the separation distance is increasing, the frequency will be shifted lower. This shifting is known as the *Doppler effect*. The ratio of observed frequency, f', to emitted frequency, f, depends on the local speed of sound, ~~a~~, and the absolute velocities of the source and observer, v_s and v_o. In Eq. 72.14, v_s is positive if the source moves away from the observer and is negative otherwise; v_o is positive if the observer moves toward the source and negative otherwise.

$$\frac{f'}{f} = \frac{c + \mathrm{v}_o}{c + \mathrm{v}_s} \qquad 72.14$$

Example 72.4

On a particular day, the local speed of sound is 1130 ft/sec (344 m/s). What frequency is heard by a stationary pedestrian when a 1200 Hz siren on an emergency vehicle moves away at 45 mph (72 kph)?

SI Solution

The observer's velocity is zero. The source's velocity is positive because the vehicle is moving away. From Eq. 72.14, the frequency heard is

$$\begin{aligned} f' &= f\left(\frac{c + \mathrm{v}_o}{c + \mathrm{v}_s}\right) \\ &= (1200\ \mathrm{Hz})\left(\frac{344\ \frac{\mathrm{m}}{\mathrm{s}} + 0\ \frac{\mathrm{m}}{\mathrm{s}}}{344\ \frac{\mathrm{m}}{\mathrm{s}} + \frac{\left(72\ \frac{\mathrm{km}}{\mathrm{h}}\right)\left(1000\ \frac{\mathrm{m}}{\mathrm{km}}\right)}{3600\ \frac{\mathrm{s}}{\mathrm{h}}}}\right) \\ &= 1134\ \mathrm{Hz} \end{aligned}$$

Customary U.S. Solution

The observer's velocity is zero. The source velocity is positive because the vehicle is moving away. Convert the source velocity from miles per hour to feet per second.

$$\mathrm{v}_s = \frac{\left(45\ \frac{\mathrm{mi}}{\mathrm{hr}}\right)\left(5280\ \frac{\mathrm{ft}}{\mathrm{mi}}\right)}{3600\ \frac{\mathrm{sec}}{\mathrm{hr}}} = 66\ \mathrm{ft/sec}$$

From Eq. 72.14, the frequency heard is

$$\begin{aligned} f' &= f\left(\frac{c + \mathrm{v}_o}{c + \mathrm{v}_s}\right) = (1200\ \mathrm{Hz})\left(\frac{1130\ \frac{\mathrm{ft}}{\mathrm{sec}} + 0\ \frac{\mathrm{ft}}{\mathrm{sec}}}{1130\ \frac{\mathrm{ft}}{\mathrm{sec}} + 66\ \frac{\mathrm{ft}}{\mathrm{sec}}}\right) \\ &= 1134\ \mathrm{Hz} \end{aligned}$$

12. ACOUSTIC IMPEDANCE

The *acoustic impedance* (*sound impedance*), Z, is a function of the pressure, p, wave velocity, v, and the surface area, A, through which the sound travels. Common units are lbf-sec/ft^5 and N·s/m^5, equivalent to rayls/m^2. The quantity vA is known as the *volume velocity*.

$$Z = \frac{p}{\mathrm{v}A} \qquad 72.15$$

The *specific acoustic impedance*, z, is

$$z = \frac{p}{\mathrm{v}} = ZA \qquad 72.16$$

The *specific acoustic impedance* (*characteristic acoustic impedance*) *of the medium* (e.g., air, water, steel), Z_0, is given by Eq. 72.17. Common units are lbf-sec/ft^3 and frequency is 1000 Hz spans the range from 707 Hz to 1414 Hz. Since musical pitch frequency ratios of 1:2 are called *octaves*, the name *octave band* has been adopted.

Center frequencies for octave bands have been standardized at 63 Hz, 125 Hz, 250 Hz, 500 Hz, 1000 Hz, 2000 Hz, 4000 Hz, and 8000 Hz.[11,12] These bands are numbered 1, 2, 3, 4, 5, 6, 7, and 8, respectively. Octave bands can also be subdivided for better frequency detail. The most common subdivision is the *one-third octave band*.

21. COMBINING MULTIPLE SOURCES

Multiple sound sources combine to produce more sound. Sound pressures from multiple correlated sources combine linearly, while sound pressures combine as sums of squares. However, multiple sound pressure levels or sound power levels expressed in decibels cannot be directly added. Two 90 dB sources do not produce a 180 dB source. For multiple sound sources, the combined power is given by Eq. 72.22. This is known as an *unweighted sum* because the individual readings are used without modification. *ASHRAE Handbook—Fundamentals* provides a table indicating the number of decibels to add to the highest decibel level when combining two sound sources. **[Combining ~~Multiple~~ Sound Levels]**

$$L_p = 10\log\sum 10^{L_i/10} \qquad 72.22$$

Using Eq. 72.22 to add or subtract the sound from two or more sources is not the same as determining the *change* in sound level. As described in Sec. 72.30, Sec. 72.31, and Sec. 72.32, the relationship between the old, new, and change in sound levels is that of a simple linear combination (i.e., addition or subtraction).

Equation 72.22 illustrates an important noise reduction principle: The noisiest source must be identified and treated before significant overall noise reduction can be achieved. Sources with sound levels more than a few decibels lower than the noisiest source make a minor contribution to the total sound level.

Example 72.6

What is the unweighted combined sound pressure from two machines, one with a sound pressure of 89 dB and the other with a sound pressure of 94 dB?

Solution

Use Eq. 72.22.

$$\begin{aligned} L_p &= 10\log\sum 10^{L_i/10} \\ &= 10\log\left(10^{89\text{ dB}/10} + 10^{94\text{ dB}/10}\right) \\ &= 95.2\text{ dB} \end{aligned}$$

22. BROAD BAND MEASUREMENTS

Octave band measurements contain enough information about the frequency content to permit calculation of the equivalent A-weighted level. The correction from Table 72.4 is made to each octave band measurement, and the corresponding levels are added using Eq. 72.22.

Table 72.4 *A-Weighting Corrections from Octave Band Analysis*

center frequency of band (Hz)	correction to be added (dB)*
31.5	–39.2
63	–26.2
125	–16.1
250	–8.6
500	–3.2
1000	0.0
2000	+1.2
4000	+1.0
8000	–1.1

*referenced to 20 μPa

23. LOUDNESS

Human hearing is not equally sensitive to pressure at all frequencies. In addition, some types of noise are more annoying than others. Subjective (i.e., perceived) noise level is called *loudness*.

There are two loudness scales: the *phon scale* and the *sone scale*. One *phon* is numerically equal to the sound pressure level in decibels at a frequency of 1000 Hz. At 1000 Hz, the values of loudness and L_p are the same.

The common loudness scale used by fan and duct manufacturers is the sone scale. By definition, one *sone* is the loudness of a 1000 Hz sound with a sound pressure of 0.02 μbar.[13] Sones are calculated from octave band sound pressure level measurements made through frequency filters in much the same way as are dBA measurements.[14]

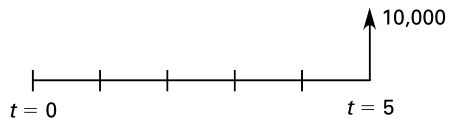
[11]A now-obsolete series of octave bands was used until the late 1950s.

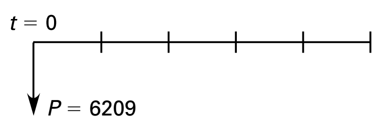
[12]Octave bands centered at 31.5 Hz and 16,000 Hz are also occasionally encountered. However, these two end bands are usually omitted, as they are at the extreme limits of most peoples' hearing abilities. For example, the A-weighted correction for 31.5 Hz is −39.2 dB. This is such a large reduction that the contribution from this band is negligible except for all but the most powerful or pure-tone sources.

[13]This is approximately the loudness of a quiet refrigerator in a quiet kitchen.

[14]However, sone values are combined and manipulated differently than decibel (dBA) values.

The factor 0.6209 would usually be obtained from the tables.





11. STANDARD CASH FLOW FACTORS AND SYMBOLS

Equation 73.2 and Eq. 73.3 may give the impression that solving engineering economic analysis problems involves a lot of calculator use, and, in particular, a lot of exponentiation. Such calculations may be necessary from time to time, but most problems are simplified by the use of tabulated values of the factors.

Rather than actually writing the formula for the compound amount factor (which converts a present amount to a future amount), it is common convention to substitute the standard functional notation of (F/P, $i\%$, n). Therefore, the future value in n periods of a present amount would be symbolically written as [Economic Analysis: Nomenclature and Definitions] [Economic Factor Conversions]

$$F = P(F/P, i\%, n) \qquad 73.4$$

Similarly, the present worth factor has a functional notation of (P/F, $i\%$, n). Therefore, the present worth of a future amount n periods in the future would be symbolically written as

$$P = F(P/F, i\%, n) \qquad 73.5$$

Values of these *cash flow* (*discounting*) *factors* are tabulated in AAA XX.XXX. There is often initial confusion about whether the (F/P) or (P/F) column should be used in a particular problem. There are several ways of remembering what the functional notations mean. [Economic Analysis: Nomenclature and Definitions]

One method of remembering which factor should be used is to think of the factors as conditional probabilities. The conditional probability of event **A** given that event **B** has occurred is written as $p\{\mathbf{A}|\mathbf{B}\}$, where the given event comes after the vertical bar. In the standard notational form of discounting factors, the given amount is similarly placed after the slash. What you want comes before the slash. (F/P) would be a factor to find F given P. [Economic Factor Conversions]

Another method of remembering the notation is to interpret the factors algebraically. The (F/P) factor could be thought of as the fraction F/P. Algebraically, Eq. 73.4 would be

$$F = P\left(\frac{F}{P}\right) \qquad 73.6$$

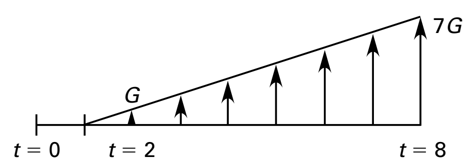
This algebraic approach is actually more than an interpretation. The numerical values of the discounting factors are consistent with this algebraic manipulation. The (F/A) factor could be calculated as (F/P) × (P/A). This consistent relationship can be used to calculate other factors that might be occasionally needed, such as (F/G) or (G/P). For instance, the annual cash flow that would be equivalent to a uniform gradient may be found from

$$A = G(P/G, i\%, n)(A/P, i\%, n) \qquad 73.7$$

Formulas for the compounding and discounting factors are contained in Table 73.1. Normally, it will not be necessary to calculate factors from the formulas. AAA XX.XXX is adequate for solving most problems. [Economic Analysis: Nomenclature and Definitions]

Example 73.3

What factor will convert a gradient cash flow ending at $t = 8$ to a future value at $t = 8$? (That is, what is the (F/G, $i\%$, n) factor?) The effective annual interest rate is 10%.



Solution

method 1:

Find the (F/G, 10%, 8) factor using Tstyleable Table 73.1. [Economic Factor Conversions]

$$\begin{aligned}(F/G, 10\%, 8) &= \frac{(1+i)^n - 1}{i^2} \\ &= \frac{(1+0.10)^8 - 1}{(0.10)^2} - \frac{8}{0.10} \\ &= 34.3589\end{aligned}$$

method 2:

The tabulated values of (P/G) and (F/P) in AAA XX.XXX can be used to calculate the factor. [Economic Factor Conversions]

$$\begin{aligned}(F/G, 10\%, 8) &= (P/G, 10\%, 8)(F/P, 10\%, 8) \\ &= (16.0287)(2.1436) \\ &= 34.3591\end{aligned}$$

Fluids

In the operation of a *centrifugal pump*, liquid flowing into the *suction side* (the *inlet*) is captured by the *impeller* and thrown to the outside of the pump casing. Within the casing, the velocity imparted to the fluid by the impeller is converted into pressure energy. The fluid leaves the pump through the *discharge line* (the *exit*). It is a characteristic of most centrifugal pumps that the fluid is turned approximately 90° from the original flow direction. (See Table 18.1.)

Table 18.1 *Generalized Characteristics of Positive Displacement and Kinetic Pumps*

characteristic	positive displacement pumps	kinetic pumps
energy transfer	~~intermittent~~	~~continuous~~
flow rate	low	high
pressure rise per stage	high	low
constant quantity over operating range	flow rate	pressure rise
self-priming	yes	no
discharge stream	pulsing	steady
works with high viscosity fluids	yes	no

3. RECIPROCATING POSITIVE DISPLACEMENT PUMPS

Reciprocating positive displacement (PD) pumps can be used with all fluids, and are useful with viscous fluids and slurries (up to about 8000 Saybolt seconds universal (SSU), when the fluid is sensitive to shear, and when a high discharge pressure is required.[1] By entrapping a volume of fluid in the cylinder, reciprocating pumps provide a fixed-displacement volume per cycle. They are self-priming and inherently leak-free. Within the pressure limits of the line and pressure relief valve and the current capacity of the motor circuit, reciprocating pumps can provide an infinite discharge pressure.[2]

There are three main types of reciprocating pumps: power, direct-acting, and diaphragm. A *power pump* is a *cylinder-operated pump*. It can be single-acting or double-acting. A *single-acting pump* discharges liquid (or takes suction) only on one side of the piston, and there is only one transfer operation per crankshaft revolution. A *double-acting pump* discharges from both sides, and there are two transfers per revolution of the crank.

Traditional reciprocating pumps with pistons and rods can be either single-acting or double-acting and are suitable up to approximately 2000 psi (14 MPa). *Plunger pumps* are only single-acting and are suitable up to approximately 10,000 psi (70 MPa).

Simplex pumps have one cylinder, *duplex pumps* have two cylinders, *triplex pumps* have three cylinders, and so forth. *Direct-acting pumps* (sometimes referred to as *steam pumps*) are always double-acting. They use steam, unburned fuel gas, or compressed air as a motive fluid.

PD pumps are limited by both their NPSH_R characteristics, acceleration head, and (for rotary pumps) slip.[3] Because the flow is unsteady, a certain amount of energy, the *acceleration head*, h_{ac}, is needed to accelerate the fluid flow each stroke or cycle. If the acceleration head needed is too large, the NPSH_R requirements may not be attainable. Acceleration head can be reduced by increasing the pipe diameter, shortening the suction piping, decreasing the pump speed, or placing a *pulsation damper* (*stabilizer*) in the suction line.[4]

Generally, friction losses with pulsating flows are calculated based on the maximum velocity attained by the fluid. Since this is difficult to determine, the maximum velocity can be approximated by multiplying the average velocity (calculated from the rated capacity) by the factors in Table 18.2.

Table 18.2 *Typical v_{max}/v_{ave} Velocity Ratios*[a,b]

pump type	single-acting	double-acting
simplex	3.2	2.0
duplex	1.6	1.3
triplex	1.1	1.1
quadriplex	1.1	1.1
quintuplex and up	1.05	1.05

[a]Without stabilization. With properly sized stabilizers, use 1.05–1.1 for all cases.

[b]Multiply the values by 1.3 for metering pumps where lost fluid motion is relied on for capacity control.

When the suction line is "short," the acceleration head can be calculated from the length of the suction line, the average velocity in the line, and the rotational speed.[5]

[1]For viscosities of SSU greater than 240, multiply SSU viscosity by 0.216 to get viscosity in centistokes.

[2]For this reason, a relief valve should be included in every installation of positive displacement pumps. Rotary pumps typically have integral relief valves, but external relief valves are often installed to provide easier adjusting, cleaning, and inspection.

[3]Manufacturers of PD pumps prefer the term *net positive inlet pressure* (NPIP) to NPSH. NPIPA corresponds to NPSH_A; NPIPR corresponds to NPSH_R. Pressure and head are related by $p = \gamma h$.

[4]Pulsation dampers are not needed with rotary-action PD pumps, as the discharge is essentially constant.

[5]With a properly designed pulsation damper, the effective length of the suction line is reduced to approximately 10 pipe diameters.

Example 23.10

What mass of nitrogen is contained in a 2000 ft^3 (57 m^3) tank at 1 atm and 70°F (21°C)?

SI Solution

First, convert to absolute temperature.

$$T = 21°\text{C} + 273° = 294\text{K}$$

From a table of ideal gas properties, the specific gas constant for nitrogen is 0.2968 kJ/kg·K. **[Thermal and Physical Properties of Ideal Gases (at Room Temperature)]**

Use the ideal gas law (Eq. 23.71), and solve for mass.

Ideal Gas

$$\begin{aligned} pV &= mRT \\ m &= \frac{pV}{RT} \\ &= \frac{\left((1\,\text{atm})\left(1.013\times 10^5\ \frac{\text{Pa}}{\text{atm}}\right)\right)(57\,\text{m}^3)}{\left(0.2968\ \frac{\text{kJ}}{\text{kg·K}}\right)(294\text{K})} \\ &= 66.1\,\text{kg} \end{aligned}$$

Customary U.S. Solution

First, convert to absolute temperature.

$$T = 70°\text{F} + 460° = 530°\text{R}$$

From a table of ideal gas properties, the specific gas constant for nitrogen is 55.16 ft-lbf/lbm-°R. **[Thermal and Physical Properties of Ideal Gases (at Room Temperature)]**

Use the ideal gas law (Eq. 23.71), and solve for mass.

Ideal Gas

$$\begin{aligned} pV &= mRT \\ m &= \frac{pV}{RT} \\ &= \frac{\left((1\ \text{atm})\left(14.7\ \frac{\text{lbf}}{\text{in}^2\text{-atm}}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2\right)\times(2000\ \text{ft}^3)}{\left(55.16\ \frac{\text{ft-lbf}}{\text{lbm-°R}}\right)(530°\text{R})} \\ &= \cancel{66.1\ \text{kg}} \end{aligned}$$

Example 23.11

A 25 ft^3 (0.71 m^3) tank contains 10 lbm (4.5 kg) of an ideal gas. The gas has a molecular weight of 44 and is at 70°F (21°C). What is the pressure of the gas?

SI Solution

Use Eq. 23.70 to calculate the specific gas constant. 8314 J/kmol·K is the universal gas constant. **[Fundamental Constants]**

$$\begin{aligned} R &= \frac{\overline{R}}{M} = \frac{8314\ \frac{\text{J}}{\text{kmol·K}}}{44\ \frac{\text{kg}}{\text{kmol}}} \\ &= 189.0\ \text{J/kg·K} \end{aligned}$$

The absolute temperature is $T = 21°\text{C} + 273° = 294\text{K}$. Use the ideal gas law (Eq. 23.71), and solve for pressure.

Ideal Gas

$$\begin{aligned} pV &= mRT \\ p &= \frac{mRT}{V} \\ &= \frac{(4.5\ \text{kg})\left(189\ \frac{\text{J}}{\text{kg·K}}\right)(294\text{K})}{(0.71\,\text{m}^3)\left(1000\ \frac{\text{Pa}}{\text{kPa}}\right)} \\ &= 352.2\,\text{kPa} \end{aligned}$$

Customary U.S. Solution

Use Eq. 23.70 to calculate the specific gas constant. 1545 ft-lbf/lbm-°R is the universal gas constant. **[Fundamental Constants]**

$$\begin{aligned} R &= \frac{\overline{R}}{M} = \frac{1545\ \frac{\text{ft-lbf}}{\text{lbmol-°R}}}{44\ \frac{\text{lbm}}{\text{lbmol}}} \\ &= 35.11\ \text{ft-lbf/lbm-°R} \end{aligned}$$

The absolute temperature is $T = 70°\text{F} + 460° = 530°\text{R}$. Use the ideal gas law Eq. 23.71), and solve for pressure.

Ideal Gas

$$\begin{aligned} pV &= mRT \\ p &= \frac{mRT}{V} \\ &= \frac{(10\ \text{lbm})\left(35.11\ \frac{\text{ft-lbf}}{\text{lbm-°R}}\right)(530°\text{R})}{25\,\text{ft}^3} \\ &= 7443\ \text{lbf/ft}^2 \end{aligned}$$

34. PROPERTIES OF IDEAL GASES

A gas can be considered to behave ideally if its pressure is very low and the temperature is much higher than its critical temperature. (Otherwise, the substance is in

30 Advanced and Alternative Power-Generating Systems

Content in blue refers to the *NCEES Handbook*.

1. INTRODUCTION

Many advanced energy technologies are maturing but in some cases uneconomical, are limited to specific applications, or are still in various stages of development. Renewables are limited by localization and capacity factor. (An installation's *capacity factor* is the actual power output over some period of time divided by the theoretical maximum output. A wind turbine's capacity factor, for example, is affected by the percentages of time the wind does not blow.) Not only are most renewables confined to specific locations, but even then, their capacity factors are often below 30%.

The 1973 energy crisis and oil embargo illuminated the need for alternative energy sources. However, with the large fossil fuel reserves in the United States, wide geographic distribution of these reserves, and the high efficiencies being achieved in combined cycle plants, it is likely that coal and natural gas will continue to be the most economic source of baseload electricity well into the future. Of all the other renewable energy sources, wind energy comes closest in price. Even so, and even with tax credit incentives (i.e., "production credits"), wind power is still 25% to 100% more expensive than coal/gas power.

~~Renewables are limited by localization and capacity factor. (An installation's capacity factor is the actual power output over some period of time divided by the theoretical maximum output. A wind turbine's capacity factor, for example, is affected by the percentages of time the wind does not blow.) Not only are most renewables confined to specific locations, but even then, their capacity factors are often below 30%. (By comparison, new coal plants have capacity factors in excess of 85%.)~~

2. SOLAR THERMAL ENERGY

Solar thermal energy arrives at the outside of the earth's atmosphere at an average rate of 433 $\text{Btu/ft}^2\text{-hr}$ ($1.366\ \text{kW/m}^2$), a value known as the *solar constant*. 40% to 70% of this energy survives absorption in and reflection from the atmosphere and reaches the earth's surface. The actual incident energy, I, sometimes referred to as *insolation*, depends on many factors, including geographic location, tilt and orientation of the receiving surface, calendar day, time of day, and weather conditions. Average values for clear days are given in maps and tabulations.[1] Some of this energy can be captured in *active solar systems* and used for space heating, domestic hot water (DHW) generation, and cooling (using heat pumps and absorption chillers).[2]

Solar thermal energy is captured in *solar collectors*. The sun's energy enters the collector and, because of the *greenhouse effect*, is trapped inside.[3] Heat is absorbed by *heat transfer fluid* pumped through tubes mounted on the *absorber plate*.[4] The tubes and absorber plate can be left uncoated, painted black, or treated with a

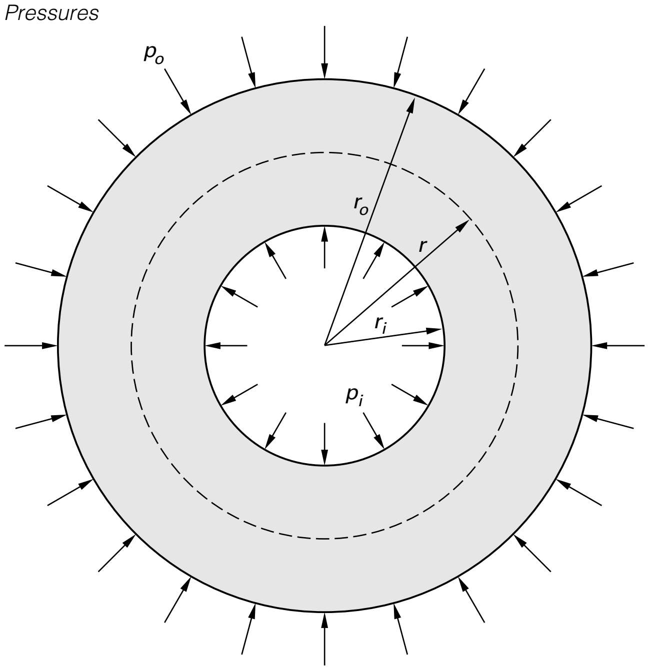
[1]Some insolation maps use units of langleys/day. A *langley* is equal to $1\ \text{cal/cm}^2$, $3.69\ \text{Btu/ft}^2$, and $41\,840\ \text{J/m}^2$.

[2]*Passive solar systems*, which include strategically oriented buildings, walls, and thermal collectors, and which rely on natural convection and conduction for storage and heat transfer, are not included in this chapter.

[3]As received, solar radiation has wavelengths of 0.2 μm to 3.0 μm. Thermal radiation reradiated from the collector plate has a wavelength of approximately 3 μm. Good covering materials have high transmittance (85% to 95%) of received radiation and significantly lower transmittance (less than 2%) of reradiated radiation. White crystal glass, low-iron tempered and sheet glasses, and tempered float glass satisfy these requirements. Polycarbonates, acrylics, and fiberglass also perform well but suffer from weathering and durability problems.

[4]Water can be used as the heat transfer medium, but it is subject to freezing, boiling, and chemical breakdown; and the system is subject to corrosion. To counteract these problems, ethylene glycol-water and glycerine-water mixtures are often used. Ethylene glycol, however, is toxic, and a heat exchanger must be used to keep the heat transfer fluid separate from domestic water. "State-of-the-art" fluids include silicones, hydrocarbon (aromatic and paraffinic) oils, and change-of-phase refrigerants (see Table 30.1).

Figure 54.6 *Cylindrical Vessel Subjected to Internal and External Pressures*



Cylindrical Pressure Vessel

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + \dfrac{r_i^2 r_o^2 (p_o - p_i)}{r^2}}{r_o^2 - r_i^2} \qquad 54.2$$

For a thick-walled vessel, the axial stress is given by Eq. 54.3. Axial stress is constant within the wall; it does not vary with the distance from the center of the vessel.

Cylindrical Pressure Vessel

$$\sigma_a = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} \qquad 54.3$$

Equation 54.1 through Eq. 54.3 can often be simplified for specific conditions. When calculating stress on the inner surface, r_i is equal to r; for stress on the outer surface, r_o is equal to r. If the stress is based only on internal pressure, $\cancel{r_o}$ is zero; if the stress is based only on external pressure, p_i is zero. For example, in calculating the tangential stress on the inner surface of a cylindrical vessel ($r_i = r$) due to internal pressure ($\cancel{r_o} = 0$), Eq. ~~54.3~~ can be simplified to Eq. 54.4.

Cylindrical Pressure Vessel

$$\sigma_t = p_i \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \right) \qquad 54.4$$

For a thin-walled cylindrical vessel, tangential and axial stresses vary only with the vessel's diameter, D, and the internal pressure, p_i. Tangential stress can be found from Eq. 54.5, and axial stress can be found from Eq. 54.6. Tangential stress is twice axial stress, so it is tangential stress that drives pressure vessel design. In a thin-walled vessel, radial stress is significantly less than both the tangential and axial stresses and is typically neglected.

Cylindrical Pressure Vessel

$$\sigma_t = \frac{p_i D}{2t} \qquad 54.5$$

$$\sigma_l = \frac{p_i D}{4t} \qquad 54.6$$

Equation 54.5 and Eq. 54.6 can also be used to calculate the stresses in circumferential and longitudinal joints[20], but cannot be used to calculate wall thickness. Wall thickness calculations are addressed in Sec. 54.25.

For a thin-walled pipe with a maximum allowable tangential stress of S, the maximum working pressure can be found from the Barlow formula.

Cylindrical Pressure Vessel

$$p = \frac{2St}{D} \qquad 54.7$$

19. CORROSION ALLOWANCE

An optional *corrosion allowance* compensates for any wall thinning expected over the lifetime of the vessel. The BPVC does not provide guidance in determining the allowance. Values will be unique to each situation and must be determined by the user considering vessel duty and corrosiveness of the contents. Generic total corrosion allowances of one-sixth of the required thickness, $^1/_{16}$ in (1.6 mm), and $^1/_8$ in (3.2 mm) are common, as is a corrosion rate of 0.005 in/yr (0.13 mm/yr), but these have no specific basis other than tradition. A corrosion allowance of zero is common for stainless steel and other nonferrous materials not subject to corrosion. An allowance of 1 mm is typical for air receivers where moisture condensation is inevitable. Usually, the maximum corrosion allowance for carbon steel is 6 mm.

Every dimension used in a formula should be a corroded dimension.[21] When calculating the thickness from a known pressure, if a formula (such as from Table 54.9 or Table 54.10) used to calculate the theoretical thickness includes a nominal radius term, r, the corrosion allowance should be added to the radius before calculating the theoretical thickness. This will account for the slight

[20]The term girth seam is sometimes used when referring to a circumferential joint.

[21]As illustrated in BPVC Sec. VIII, Div. 1, App. L (e.g., Ex. L.9.2.1), this admonition includes adding the corrosion allowance to the nominal radius as well as to the resulting required thickness. The effect of increasing the radius is small, and many authorities simply add the corrosion allowance to the thickness as a final step.

procedure that accounts for this aspect of gear design. The general procedure is to keep the limit wear load (capacity) greater than the dynamic load. The dynamic load is calculated from the transmitted tangential force, F_t, with corrections for *errors in action* (e.g., inaccuracies in tooth form, cutting, and spacing, mounting misalignments, and rotational inertia).[23]

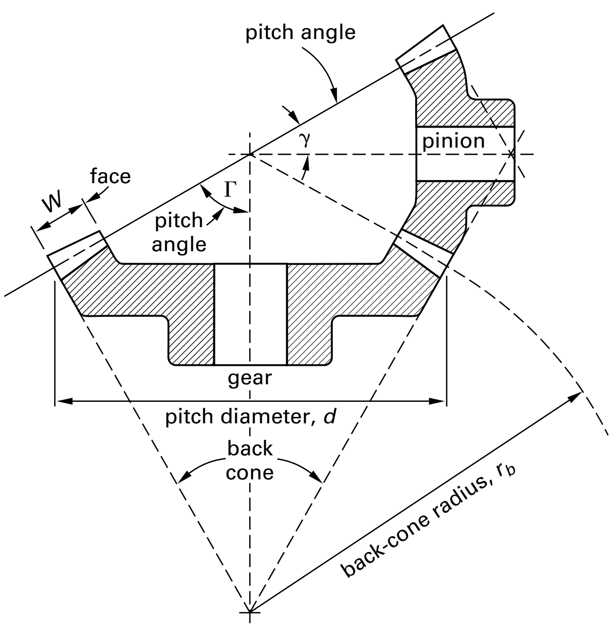
26. BEVEL GEARS

Bevel gear sets are used with intersecting shafts. Gears can be straight or spiral.[24] Figure 53.11 illustrates a straight bevel gear set. The most common shaft angle is 90°, although any angle can be accommodated. The circular pitch and pitch diameter are as defined for spur gears. The *pitch angles*, γ and Γ, are the angles of the forward *pitch cones* projected back to the apex.

$$\tan\gamma = \frac{N_{\text{pinion}}}{N_{\text{gear}}} \qquad 53.72$$

$$\tan\Gamma = \frac{N_{\text{gear}}}{N_{\text{pinion}}} \qquad 53.73$$

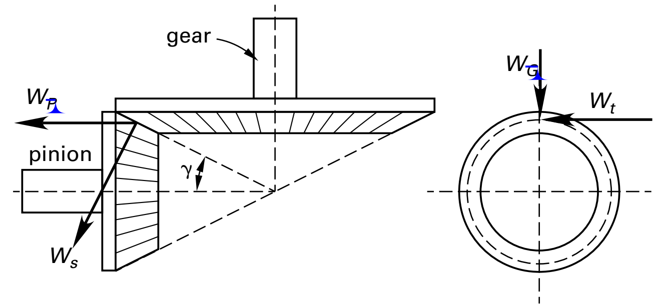
Figure 53.11 *Straight Bevel Gear Set*



Bevel gears experience a *separation force* (*thrust force*) away from the apex. Installation (choice of bearings) must be designed to ensure that gears are not thrown out of alignment as they are loaded. A force analysis of bevel gears depends on the pressure angle, ϕ, and the pinion pitch cone angle, γ.

The forces on a bevel gear set are shown in Fig. 53.12 and can be calculated from the following equations, which assume the pinion, P, drives the gear, G. T is the torque on the pinion shaft, W_s is the separating force, W_a is the pinion thrust, and W_r is the gear thrust. Equation 53.53 relates the torque to power transfer and can be used calculate the tangential force on the pinion, W_t. The pitch diameter, D_p, is twice the mean radius, r_{av}.

Figure 53.12 *Forces on Bevel Gear Set*



$$T = r_{\text{av}} W_t = \frac{D_p W_t}{2} \qquad 53.74$$

$$W_s = W_t \tan\phi \qquad 53.75$$

Bevel Gears

$$W_a = W_t \tan\phi \sin\gamma \qquad 53.76$$

Bevel Gears

$$W_r = W_t \tan\phi \cos\gamma \qquad 53.77$$

27. WORM GEAR SETS

Worm drives, consisting of a *worm* and a *worm gear*, are used to turn noncoplanar shafts oriented at right angles with high speed ratios (e.g., 2:1 or 3:1). (See Fig. 53.13.) Worm drives are ordinarily irreversible: A worm will turn a gear, but a gear will not turn a worm. Therefore, worm sets are normally self-locking, although this characteristic should not be depended on for safe operation.

Worms can have a single thread, but they usually have more (as many as six) threads. The number of threads (teeth) is known as the *lead ratio*, LR (N_W). A double-threaded worm has a lead ratio of 2:1 and so on. When a double-threaded (triple-threaded, etc.) worm is rotated through a complete revolution, two (three, etc.) worm pitches pass by a fixed point. When in mesh, one revolution of the worm moves two (three, etc.) pitches of the

[23]The *Buckingham equation* was widely used for many years to calculate dynamic effects until the AGMA equations were developed.
[24]*Hypoid gears* are essentially spiral bevel gears for shafts that do not intersect.

and the rotational speed is high, the *Petroff equation* can be used to find the frictional torque. In Eq. 53.141, r is the bearing radius and L is its length.

Ball/Roller Bearing Selection

$$T_{f,\text{N}\cdot\text{m}} = \frac{4\pi^2 r^3 L\mu_{\text{Pa}\cdot\text{s}} N_{\text{rps}}}{c} \quad \text{[SI]} \qquad \textit{53.141(a)}$$

$$T_{f,\text{in-lbf}} = \frac{4\pi^2 r^3 L\mu_{\text{reyns}} N_{\text{rps}}}{c} \quad \text{[U.S.]} \qquad \textit{53.141(b)}$$

The effective coefficient of friction is

$$f = 2\pi^2 \frac{\mu N}{P}\frac{r}{c} = \frac{2\pi^2 \mu_{\text{reyns}} N_{\text{rps}} D}{p c_d} \qquad \textit{53.142}$$

The frictional horsepower, $P_{f,\text{hp}}$, can be found from frictional torque by using Eq. 53.37. Frictional heating varies with the square of the rotational speed. The usual range of operating temperatures is 140°F to 160°F (60°C to 70°C). Most lubricants start to deteriorate above 200°F (90°C). Frictional heating, q_f, is dissipated in an oil cooler or through contact with a cooler surface. The specific heat for petroleum oils is approximately 0.42 Btu/lbm-°F to 0.49 Btu/lbm-°F (1.8 kJ/kg·K to 2.0 kJ/kg·K).

$$q_f = P_{f,\text{hp}} = \dot{m} c_p \Delta T \qquad \textit{53.143}$$

The ratio of loaded to unloaded friction torque can be correlated to the load number. The *load number* is calculated from Eq. 53.144.

$$N_L = \left(\frac{p}{\mu_{\text{reyns}} n_{\text{rps}}}\right)\left(\frac{c_d}{L}\right)^2 \qquad \textit{53.144}$$

Example 53.2

SAE 20 oil at 150°F (65°C) bulk temperature is used in a lightly loaded journal bearing. The bearing has a length of 5 in (12.7 cm) and a radius of 1.5 in (3.81 cm). The shaft turns at 1800 rpm. The ~~diametral~~ clearance is 0.005 in (0.127 mm). What is the frictional power?

SI Solution

From Fig. 53.24, the viscosity of SAE 20 oil at 65°C is approximately 2.5×10^{-6} reyns. Combining Eq. 53.131 and Eq. 53.132,

$$\begin{aligned} \mu_{\text{Pa}\cdot\text{s}} &= \frac{(6.89 \times 10^6)\mu_{\text{reyns}}}{1000} \\ &= \frac{(6.89 \times 10^6)(2.5 \times 10^{-6}\ \text{reyns})}{1000} \\ &= 0.017225\ \text{Pa}\cdot\text{s} \end{aligned}$$

From Eq. 53.141, the frictional torque is

Ball/Roller Bearing Selection

$$\begin{aligned} T_{\text{N}\cdot\text{m}} &= \frac{4\pi^2 r^3 L\mu_{\text{Pa}\cdot\text{s}} N_{\text{rps}}}{c} \\ &= \frac{4\pi^2(0.0381\ \text{m})^3(0.127\ \text{m}) \times (0.017225\ \text{Pa}\cdot\text{s})\left(1800\ \frac{\text{rev}}{\text{min}}\right)}{\left(60\ \frac{\text{sec}}{\text{min}}\right)\left(\frac{0.127\ \text{mm}}{1000\ \frac{\text{mm}}{\text{m}}}\right)} \\ &= 1.1283\ \text{N}\cdot\text{m} \end{aligned}$$

Equation 53.37 gives the frictional power.

$$\begin{aligned} P_{\text{kW}} &= \frac{T_{\text{N}\cdot\text{m}} n_{\text{rpm}}}{9549} \\ &= \frac{(1.1283\ \text{N}\cdot\text{m})\left(1800\ \frac{\text{rev}}{\text{min}}\right)}{9549} \\ &= 0.2127\ \text{kW} \end{aligned}$$

Customary U.S. Solution

From Fig. 53.24, the viscosity of SAE 20 oil at 150°F is approximately 2.5×10^{-6} reyns. From Eq. 53.141, the frictional torque is

Ball/Roller Bearing Selection

$$\begin{aligned} T_{f,\text{in-lbf}} &= \frac{4\pi^2 r^3 L\mu_{\text{reyns}} N_{\text{rps}}}{c} \\ &= \frac{\pi^2(\cancel{4}\ \text{in})^3(5\ \text{in})\left(2.5 \times 10^{-6}\ \frac{\text{lbf-sec}}{\text{in}^2}\right)\left(1800\ \frac{\text{rev}}{\text{min}}\right)}{\left(60\ \frac{\text{sec}}{\text{min}}\right)(0.005\ \text{in})} \\ &= \cancel{79.94}\ \text{in-lbf} \end{aligned}$$

Equation 53.37 gives the frictional power.

Shaft-Horsepower Relationship and Force-Horsepower Relationship

$$\begin{aligned} \text{HP} &= \frac{T_{\text{in-lbf}} n_{\text{rpm}}}{63{,}025} = \frac{(\cancel{79.94}\ \text{in-lbf})\left(1800\ \frac{\text{rev}}{\text{min}}\right)}{63{,}025} \\ &= \cancel{2.283}\ \text{hp} \end{aligned}$$