63.2% in one time constant (i.e., will be reduced to 36.8% of its original value, $T_0 - T_\infty$).

$$\tau = \frac{c_p \rho L_c}{h} = \frac{c_p \rho V}{hA} \qquad 34.61$$

Example 34.4

A 0.03125 in (0.8 mm) diameter copper wire is heated by a short circuit to 300°F (150°C) before a slow-blow fuse burns out and all heating ceases. The ambient temperature is 100°F (38°C), and the film coefficient on the wire is 1.65 Btu/hr-ft^2-°F (9.37 W/m^2·K). Use the following copper properties to determine how long it will take for the wire temperature to drop to 120°F (49°C).

380 J/kg-K

conductivity, k	224 Btu-ft/hr-ft^2-°F	(388 W/m·K)
specific heat, c_p	0.091 Btu/lbm-°F	(~~388~~ J/kg·K)
density, ρ	558 lbm/ft^3	(8940 kg/m^3)

SI Solution

The characteristic length of the wire is half of the radius.

$$L_c = \frac{r}{2} = \frac{d}{4} = \frac{0.0008 \text{ m}}{4} = 0.0002 \text{ m}$$

The Biot number is

$$\text{Bi} = \frac{hL_c}{k} = \frac{\left(9.37 \ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)(0.0002 \text{ m})}{388 \ \frac{\text{W}}{\text{m}\cdot\text{K}}} = 4.83 \times 10^{-6} \quad [< 0.1]$$

The Fourier number is

$$\text{Fo} = \frac{kt}{\rho c_p L_c^2} = \frac{\left(388 \ \frac{\text{W}}{\text{m}\cdot\text{K}}\right)t}{\left(8940 \ \frac{\text{kg}}{\text{m}^3}\right)\left(380 \ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(0.0002 \text{ m})^2} = 2855t \quad [t \text{ in seconds}]$$

From Eq. 34.53,

$$T_t = T_\infty + (T_0 - T_\infty)e^{-\text{BiFo}}$$

$$49^\circ\text{C} = 38^\circ\text{C} + (150^\circ\text{C} - 38^\circ\text{C})e^{-(4.83\times 10^{-6})(2855t)}$$

$$0.0982 = e^{-0.0138t}$$

$$t = \frac{-1}{0.0138} \ln 0.0982 = 168 \text{ s} \quad (0.047 \text{ h})$$

Customary U.S. Solution

From Sec. 34.11, the characteristic length of the wire is half of the radius.

$$L_c = \frac{r}{2} = \frac{d}{4} = \frac{0.03125 \text{ in}}{(4)\left(12 \ \frac{\text{in}}{\text{ft}}\right)} = 0.00065 \text{ ft}$$

The Biot number is

$$\text{Bi} = \frac{hL_c}{k} = \frac{\left(1.65 \ \frac{\text{Btu}}{\text{hr-ft}^2\text{-°F}}\right)(0.00065 \text{ ft})}{224 \ \frac{\text{Btu}}{\text{hr-ft}^2\text{-°F}}} = 4.79 \times 10^{-6} \quad [< 0.1]$$

The Fourier number is

$$\text{Fo} = \frac{kt}{\rho c_p L_c^2} = \frac{\left(224 \ \frac{\text{Btu-ft}}{\text{hr-ft}^2\text{-°F}}\right)t}{\left(558 \ \frac{\text{lbm}}{\text{ft}^3}\right)\left(0.091 \ \frac{\text{Btu}}{\text{lbm-°F}}\right)(0.00065 \text{ ft})^2} = 1.044 \times 10^7 t \quad [t \text{ in hours}]$$

From Eq. 34.53,

$$T_t = T_\infty + (T_0 - T_\infty)e^{-\text{BiFo}}$$

$$120^\circ\text{F} = 100^\circ\text{F} + (300^\circ\text{F} - 100^\circ\text{F})e^{-(4.79\times 10^{-6})(1.044\times 10^7 t)}$$

$$0.10 = e^{-50t}$$

$$t = \left(\frac{-1}{50}\right)\ln 0.1 = 0.046 \text{ hr} \quad (166 \text{ sec})$$

41. GRAPHICAL SOLUTIONS TO TRANSIENT HEAT TRANSFER

If the Biot number is greater than approximately 0.1, the internal resistance of the object cannot be disregarded. If Fo $\geq$ 0.2 and the geometry is sufficiently simple, a graphical approach can be used. Graphs for transient heat flow problems are available for simple shapes, including *semi-infinite solids* (i.e., an object that is infinite in one direction only), spheres, large (infinite) slabs, and long (infinite) cylinders.

Graphs for solving transient heat transfer problems are known as *temperature-time charts*. (See App. 34.F, App. 34.G, and App. 34.H.) *Heisler charts* exhibit similar information for a particular point in the solid, but use a logarithmic scale to cover a greater range of values (in particular, longer time periods).[27] (See App. 34.I, App. 34.J, and App. 34.K.)

[27]The charts are named after H. P. Heisler, who published his original graphical methods in the *ASME Transactions* in 1947.