(i.e., hydropneumatic). *Hydropneumatic accumulators*, where pistons, diaphragms, or bladders separate the hydraulic fluid from the gas, are (by far) the most common.[21,22] Most accumulators are high-pressure tanks and, as such, should conform to ASME's *Code for Unfired Pressure Vessels*.

The volume of hydraulic fluid released by or captured in an accumulator is equal to the change in volume of the compressed gas. Compression and expansion of the gas in an accumulator is governed by standard thermodynamic principles. However, calculation of volumetric changes is complicated by the speed of the process, since the gas may heat or cool during the volume change.

The expression for polytropic processes, Eq. 19.16, is the most useful. (In Eq. 19.16, pressures should be strictly absolute pressures. However, the accumulator pressures are so high that use of gage pressures is a common practice.) V_{charged} is the volume of the gas in the accumulator when it is charged with hydraulic fluid; $V_{\text{discharged}}$ is the gas volume after the fluid discharge.

$$p_{\text{charged}} V^n_{\text{charged}} = p_{\text{discharged}} V^n_{\text{discharged}} \qquad 19.16$$

$$V_{\text{discharged}} = V_{\text{charged}} + \text{discharge} \qquad 19.17$$

Table 19.3 illustrates the variation in the polytropic exponent, n, with the charge and discharge rates. If the discharge is rapid, the process will be adiabatic, and $n = k$ (the ratio of specific heats). If it is very slow, then the process will be isothermal, and $n = 1$.

Table 19.3 *Typical Polytropic Exponents for Accumulator Sizing*

time for discharge or charging	diatomic gas[a]	monatomic gas[b]
< 1 min	1.4	1.7
2	1.3	1.5
3	1.15	1.25
> 3 min	1.0	1.0

[a]Nitrogen, air, etc.
[b]Helium, argon, etc.

The *precharge gas pressure* is the pressure in the accumulator when it is completely filled with gas and is empty of hydraulic fluid. Ideally, the precharge pressure would be the minimum system pressure. However, rapid discharge of the gas is an adiabatic (or semi-adiabatic) process, and the gas will cool to below its original system temperature, decreasing the pressure. Some of the accumulator fluid would not discharge. Therefore, the precharge pressure must be higher than the minimum system pressure.

[21]Reactive gases, such as hydrogen and oxygen, should never be used as accumulator gases.
[22]Spring and weight-loaded accumulators are much rarer.

After discharge, the accumulator will slowly warm back up to the system temperature in a constant-volume process.

$$\frac{p_{\text{discharged,cold}}}{T_{\text{discharged,cold}}} = \frac{p_{\text{discharged,warm}}}{T_{\text{discharged,warm}}} \qquad 19.18$$

The slow increase in temperature can also be considered to be an isothermal compression from the charged condition.

$$p_{\text{charged,warm}} V_{\text{charged,warm}} = p_{\text{discharged,warm}} V_{\text{discharged,warm}} \qquad 19.19$$

Since the accumulator has a rigid body, the discharged volume is equal to the total accumulator volume, regardless of the temperature or pressure in the discharged accumulator. Equation 19.17 and Eq. 19.19 can be combined to give a direct expression for the precharge pressure.

$$\begin{aligned} p_{\text{precharge}} = p_{\text{precharged,warm}} &= \frac{p_{\text{charged,warm}} V_{\text{charged,warm}}}{V_{\text{accumulator}}} \\ &= \frac{p_{\text{charged,warm}}(V_{\text{accumulator}} - \text{discharge})}{V_{\text{accumulator}}} \end{aligned} \qquad 19.20$$

Example 19.2

with air

An accumulator must supply 250 in^3 of fluid at 2000 psi minimum pressure. When charged with air, the maximum accu-mulator pressure cannot exceed 3000 psi. Discharge is assumed to be adiabatic. What are (a) the accumulator size, and (b) the required precharge pressure?

Solution

(a) From Eq. 19.17, the charged accumulator gas volume is

$$V_{\text{charged}} = V_{\text{discharged}} - 250 \text{ in}^3$$

The gas starts out compressed at 3000 psi with volume $V_{\text{warm,charged}}$. It expands to volume $V_{\text{cold,discharged}}$ in an adiabatic process. Since the process is adiabatic, $n = k = 1.4$. The pressure after the discharge is 2000 psi. Use Eq. 19.16 and Eq. 19.17.

$$p_{\text{charged}} V^n_{\text{charged}} = p_{\text{discharged}} V^n_{\text{discharged}}$$

$$V_{\text{discharged}} = \left(V_{\text{discharged}} - 250 \text{ in}^3\right)\left(\frac{3000 \ \frac{\text{lbf}}{\text{in}^2}}{2000 \ \frac{\text{lbf}}{\text{in}^2}}\right)^{1/1.4}$$

Rearranging terms and solving, the total accumulator volume is

$$V_{\text{accumulator}} = V_{\text{discharged}} = 994 \text{ in}^3$$

half of the compressive yield strength of the column material.

$$\sigma_e = \frac{F_e}{A} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2} \quad [\sigma_e < \tfrac{1}{2}S_y] \qquad 53.8$$

Add:
1/2
see equation 53.8

The quantity L/r is known as the *slenderness ratio*. Long columns have high slenderness ratios. The smallest slenderness ratio for which Eq. 53.8 is valid, found by setting $\sigma_e = S_y$, is the *critical slenderness ratio*. Typical critical slenderness ratios range from 80 to 120. The critical slenderness ratio becomes smaller as the compressive yield strength increases.

L is the longest unbraced column length. If a column is braced against buckling at some point between its two ends, the column is known as a *braced column*, and L will be less than the full column height. Columns with rectangular cross sections have two radii of gyration, r_x and r_y, and so will have two slenderness ratios. The largest slenderness ratio will govern the design.

Columns do not always have frictionless or pinned ends. Often, a column will be fixed ("clamped," "built in," etc.) at its top and base. In such cases, the *effective length*, L', must be used in place of L in Eq. 53.7 and Eq. 53.8.

$$L' = KL \qquad 53.9$$

Equation 53.8 becomes Eq. 53.10.

$$\sigma_e = \frac{F_e}{A} = \frac{\pi^2 E}{\left(\frac{L'}{r}\right)^2} \quad [\sigma_e < \tfrac{1}{2}S_y] \qquad 53.10$$

K is the *end restraint coefficient*, which varies from 0.5 to 2.0 according to Table 53.1. For most real columns, the design values of K should be used since infinite stiffness of the supporting structure is not achievable.

Euler's curve for columns, line BCD in Fig. 53.1, is generated by plotting the *Euler stress* (see Eq. 53.8) versus the slenderness ratio. Since the material's compressive yield strength cannot be exceeded, a horizontal line, AC, is added to limit applications to the region below. Theoretically, members with slenderness ratios less than $(\mathrm{SR})_\mathrm{C}$ could be treated as pure compression members. However, this is not done in practice.

Defects in materials, errors in manufacturing, inabilities to achieve theoretical end conditions, and eccentricities frequently combine to cause column failures in the region around point C. This region is excluded by designers.

Table 53.1 *Theoretical End Restraint Coefficients, K*

illus.	end conditions	ideal	recommended for design
(a)	both ends pinned	1	1.0*
(b)	both ends built in	0.5	0.65*–0.90
(c)	one end pinned, one end built in	0.707	0.80*–0.90
(d)	one end built in, one end free	2	2.0–2.1*
(e)	one end built in, one end fixed against rotation but free	1	1.2*
(f)	one end pinned, one end fixed against rotation but free	2	2.0*

*AISC values

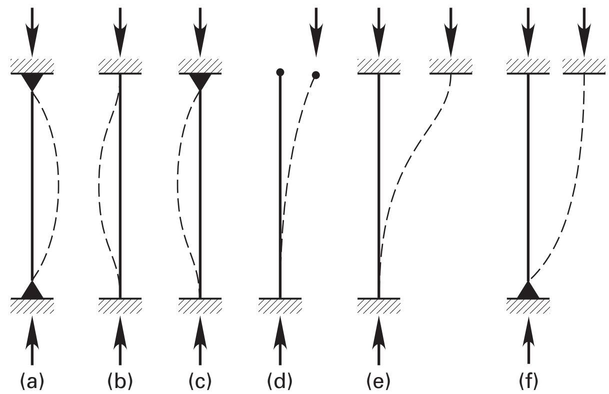
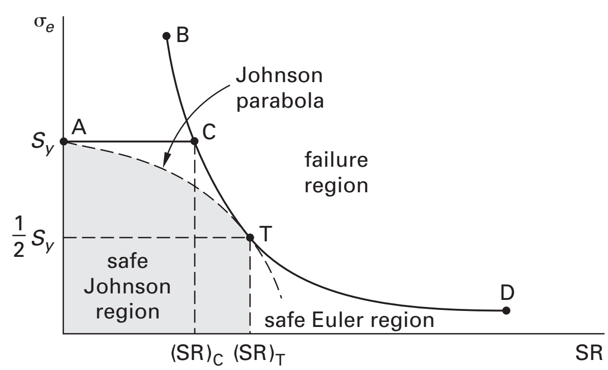


Figure 53.1 *Euler's Curve*



The empirical *Johnson procedure* used to exclude the failure area is to draw a parabolic curve from point A through a tangent point T on the Euler curve at a stress of $\frac{1}{2}S_y$. The corresponding value of the slenderness ratio is

$$(\mathrm{SR})_\mathrm{T} = \sqrt{\frac{2\pi^2 E}{S_y}} \qquad 53.11$$

The angle of contact, ϕ, for open belts depends on the radii of the two pulleys and the center-to-center distance, c.

$$\phi = \pi + 2\sin^{-1}\left(\frac{R-r}{c}\right) \quad \text{[large pulley]} \qquad 54.81$$

$$\phi = \pi - 2\sin^{-1}\left(\frac{R-r}{c}\right) \quad \text{[small pulley]} \qquad 54.82$$

Equation 54.83 and Eq. 54.84 approximate the belt lengths.

$$L_{\text{open drive}} = 2c + \left(\frac{\pi}{2}\right)(2R+2r) + \frac{(2R-2r)^2}{4c} \qquad 54.83$$

$$L_{\text{crossed drive}} = 2c + \left(\frac{\pi}{2}\right)(2R+2r) + \frac{(2R+2r)^2}{4c} \qquad 54.84$$

Ultimately, a belt is limited by the maximum tension it can support. From Eq. 54.79, with $F_2 = 0$ and $F_1 = 2F_i$, the maximum power that can be transmitted is approximately

$$P_{\text{kW}} = \frac{F_i \text{v}_{\text{m/s}}}{500} \quad \text{[SI]} \qquad 54.85(a)$$

$$P_{\text{hp}} = \frac{F_i \text{v}_{\text{ft/min}}}{16{,}500} \quad \text{[U.S.]} \qquad 54.85(b)$$

Manufacturers rate belt materials in terms of the horsepower (per unit width) that can be transmitted at various speeds. The rated power is derated by various factors. In Eq. 54.86, C_p is the pulley correction factor, C_v is the velocity correction factor, and K is the service factor. F_a is the allowable belt tension as specified by the belt manufacturer. Tables of allowable belt tension derating factors are included in most engineering handbooks.

$$P_{\text{kW}} = \frac{C_p C_\text{v} F_a \text{v}_{\text{m/s}}}{500 K_s} \quad \text{[SI]} \qquad 54.86(a)$$

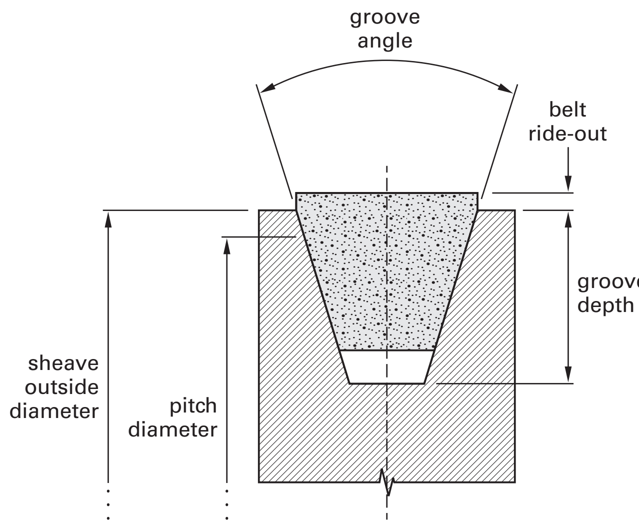
$$P_{\text{hp}} = \frac{C_p C_\text{v} F_a \text{v}_{\text{ft/min}}}{16{,}500 K_s} \quad \text{[U.S.]} \qquad 54.86(b)$$

29. V-BELTS

Design of V-belt drives is largely dependent on use of manufacturers' literature and tables from engineering handbooks. Sheave diameters are understood to be the diameter of the pitch diameters, as needed in determining the velocity ratios. V-belt cross sections have been standardized and are designated as A, B, C, D, or E and the inside diameter. Thus, a B100 V-belt would have a standard B cross section and an inside ~~diameter~~ length of 100 in.

A cross section of a V-belt and a sheave groove is shown in Fig. 54.15.

Figure 54.15 Cross Section of V-Belt and Sheave Groove



As with flat belts, the power that can be transmitted is derated by various factors, including a *service factor* that accounts for the type of prime mover and the nature of the load. In some cases, the arc of contact may be considered in derating the belt.

The number of belts is calculated from the transmitted power, the rated power per belt, and a contact arc factor (needed when the contact angle is less than 180°).

The center-to-center distance should be between 70% and 200% of the sum of the two sheave diameters. Belt length (referred to as *pitch length* and *effective length*) can be calculated in the same manner as for open flat belts. The length of a known belt can also be calculated by adding a small correction to the inside diameter of the belt. Depending on the belt type, those corrections are: A, 1.3 in; B, 1.8 in; C, 2.9 in; D, 3.3 in; and E, 4.5 in.

30. TIMING BELTS

The function of a *timing belt* (also referred to as a *synchronous belt*, *serpentine belt*, *power transmission belt*, or *timing chain*) includes maintaining the phase (synchronization, angle relationship, alignment, etc.) between the connected shafts as well as transmitting power. (See Fig. 54.16.) It is common to use timing belts in modern vehicle engines to drive overhead camshafts, usually at half the speed of the crankshaft. Belts can also be used to run water pumps and other accessories, and the flat back surface may be used for applications that do not require exact timing. Timing belts are also used in linear positioning applications as well as in motorcycle drives.

Pulleys (also referred to as *sprockets* and *gears*) are manufactured from aluminum, steel, nylon, and acetal, or a combination of two. *Idler pulleys* are used to guide belts into proper orientation, while tensioner pulleys maintain belt tension.