9 Integral Calculus

PRACTICE PROBLEMS

1. Find the indefinite integrals.

(a) $\int \sqrt{1-x}\,dx$

(A) $-\frac{2}{3}(1-x)^{3/2}+C$

(B) $-\frac{1}{2}(1-x)^{-1/2}+C$

(C) $\frac{3}{2}(1-x)^{3/2}+C$

(D) $2(1-x)^{3/2}+C$

(b) $\int \frac{x}{x^2+1}\,dx$

(A) $\frac{1}{x}\ln|(x^2+1)|+C$

(B) $\frac{1}{4}\ln|(x^2+1)|+C$

(C) $\frac{1}{2}\ln|(x^2+1)|+C$

(D) $\ln|(x^2+1)|+C$

(c) $\int \frac{x^2}{x^2+x-6}\,dx$

(A) $\ln|(x+3)|+\frac{4}{5}\ln|(x+2)|+C$

(B) $\ln|(x-3)|+\frac{4}{5}\ln|(x-2)|+C$

(C) $x-\frac{5}{9}\ln|(x-3)|+\frac{4}{5}\ln|(x-2)|+C$

(D) $x-\frac{9}{5}\ln|(x+3)|+\frac{4}{5}\ln|(2-x)|+C$

2. Calculate the definite integrals.

(a) $\int_1^3 (x^2+4x)\,dx$

(A) $16\frac{1}{2}$

(B) $24\frac{2}{3}$

(C) 27

(D) $42\frac{1}{3}$

(b) $\int_{-2}^{2} (x^3+1)\,dx$

(A) –4

(B) –2

(C) 0

(D) 4

(c) $\int_1^2 (4x^3-3x^2)\,dx$

(A) 8

(B) 16

(C) 24

(D) 32

3. Find the area bounded by $x=1$, $x=3$, $y+x+1=0$, and $y=6x-x^2$.

(A) $7\frac{1}{2}$

(B) $13\frac{1}{3}$

(C) $21\frac{1}{3}$

(D) $25\frac{1}{2}$

(b) The definite integral is

$$\int_{-2}^{2} (x^3+1)\,dx = \left(\frac{x^4}{4}+x\right)\Bigg|_{-2}^{2}$$

$$= \frac{(2)^4}{4} + 2 - \left(\frac{(-2)^4}{4} + (-2)\right)$$

$$= \boxed{4}$$

The answer is (D).

(c) The definite integral is

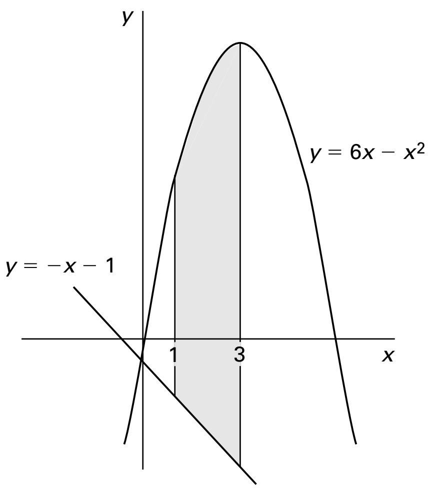
$$\int_{1}^{2} (4x^3 - 3x^2)\,dx = (x^4 - x^3)\Big|_{1}^{2}$$

$$= (2)^4 - (2)^3 - \big((1)^4 - (1)^3\big)$$

$$= \boxed{8}$$

The answer is (A).

3. The bounded area is shown.



$$\text{area} = \int_{1}^{3} \big((6x - x^2) - (-x-1)\big)\,dx$$

$$= \int_{1}^{3} (-x^2 + 7x + 1)\,dx$$

$$= \left(-\frac{x^3}{3} + \tfrac{7}{2}x^2 + x\right)\Bigg|_{1}^{3}$$

$$= -\frac{(3)^3}{3} + \left(\frac{7}{2}\right)(3)^2 + 3 - \left(-\frac{(1)^3}{3} + \left(\frac{7}{2}\right)(1)^2 + 1\right)$$

$$= \boxed{21\tfrac{1}{3}}$$

The answer is (C).

4. (a) Divide the circular internal area of the pipe into small annular rings. The radius at the ring is r; the differential thickness of the ring is dr; the differential area of the ring is $dA = 2\pi r\,dr$; and the velocity is $\mathrm{v}(r)$. The volumetric flow rate through the annular ring is

$$dQ = \mathrm{v}(r)\,dA = 2\pi r \mathrm{v}_{\max}\left(1 - \left(\frac{r}{R}\right)^2\right)dr$$

$$Q = \int_{r=0}^{r=R} 2\pi r \mathrm{v}_{\max}\left(1 - \left(\frac{r}{R}\right)^2\right)dr$$

$$= 2\pi \mathrm{v}_{\max} \int_{r=0}^{r=R} \left(r - \frac{r^3}{R^2}\right)dr$$

$$= 2\pi \mathrm{v}_{\max}\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)\Bigg|_0^R$$

$$= 2\pi \mathrm{v}_{\max}\left(\frac{R^2}{2} - \frac{R^4}{4R^2} - 0 - 0\right)$$

$$= \boxed{\frac{\pi \mathrm{v}_{\max} R^2}{2}}$$

The answer is (A)

(b) The average velocity is

$$\bar{\mathrm{v}} = \frac{Q}{A} = \frac{\dfrac{\pi \mathrm{v}_{\max} R^2}{2}}{\pi R^2} = \boxed{\frac{\mathrm{v}_{\max}}{2}}$$

The answer is ~~(A)~~. (D)

5. (a) For waveform (a),

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,dt = \frac{1}{\pi}\int_0^{\pi} f(t)\,dt$$

$$= \frac{1}{\pi}\left(r\left(\frac{\pi}{2}\right) + (-3r)\left(\frac{\pi}{2}\right)\right)$$

$$= \boxed{-r}$$

The answer is (A).

(b) For waveform (b),

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,dt = \frac{1}{\pi}\int_0^{\pi} f(t)\,dt$$

$$= \left(\frac{1}{\pi}\right)\left(\tfrac{1}{2}\pi h\right)$$

$$= \boxed{\frac{h}{2}}$$

The answer is (B).

SI Solution

Use Eq. 41.21 to calculate the friction pressure at the reduced flow rate.

$$\text{FP}_2 = \text{FP}_1\left(\frac{Q_2}{Q_1}\right)^2 = (1\ \text{kPa})\left(\frac{3700\ \frac{\text{L}}{\text{s}}}{4700\ \frac{\text{L}}{\text{s}}}\right)^2 = 0.619\ \text{kPa}$$

The dynamic loss in the duct at the reduced flow rate is 0.619 kPa. However, the pressure in the duct after the fan includes the terminal pressure of 25–75 Pa. The duct pressure will be 0.64–0.69 kPa. Therefore, the total pressure in the duct after the fan will be $\boxed{0.68\ \text{kPa.}}$

The answer is (D).

3. *Customary U.S. Solution*

The air horsepower is given by Eq. 41.13(b).

$$\text{AHP}_1 = \frac{Q_{\text{ft}^3/\text{min}}(\text{TP}_{\text{in wg}})}{6356} = \frac{\left(40\ \frac{\text{ft}^3}{\text{min}}\right)(0.5\ \text{in wg})}{6356\ \frac{\text{in-ft}^3}{\text{hp-min}}}$$
$$= 3.147\times10^{-3}\ \text{hp}$$

In order to predict the performance of a dynamically similar fan, use Eq. 41.28.

$$\frac{\text{AHP}_1}{\text{AHP}_2} = \left(\frac{D_1}{D_2}\right)^5\left(\frac{n_1}{n_2}\right)^3\left(\frac{\gamma_1}{\gamma_2}\right)$$
$$\text{AHP}_2 = \text{AHP}_1\left(\frac{D_2}{D_1}\right)^5\left(\frac{n_2}{n_1}\right)^3\left(\frac{\gamma_2}{\gamma_1}\right)$$
$$\frac{D_2}{D_1} = \frac{1}{\frac{1}{8}} = 8$$
$$\frac{n_2}{n_1} = \frac{1}{2}$$

At standard air conditions, $\rho_1 = 0.075\ \text{lbm/ft}^3$. Use standard atmospheric data from App. 25.E for 5000 ft altitude.

$$\rho_{5000\ \text{ft}} = \rho_2 = \frac{p}{RT} = \frac{\left(12.225\ \frac{\text{lbf}}{\text{in}^2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2}{\left(53.3\ \frac{\text{ft-lbf}}{\text{lbm-°R}}\right)(500.9\text{°R})}$$
$$= 0.06594\ \text{lbf/ft}^3$$

$$\frac{\gamma_2}{\gamma_1} = \frac{\frac{\rho_2 g}{g_c}}{\frac{\rho_1 g}{g_c}} = \frac{\rho_2}{\rho_1} = \frac{0.06594\ \frac{\text{lbm}}{\text{ft}^3}}{0.075\ \frac{\text{lbm}}{\text{ft}^3}} = 0.8792$$

$$\text{AHP}_2 = \left(3.147\times10^{-3}\ \text{hp}\right)(8)^5\left(\tfrac{1}{2}\right)^3(0.8792)$$
$$= \boxed{11.33\ \text{hp}\quad(11\ \text{hp})}$$

The answer is (A).

SI Solution

The air power is given by Eq. 41.13(a).

$$\text{AkW}_1 = \frac{Q_{\text{L/s}}(\text{TP}_{\text{Pa}})}{10^6}$$
$$= \frac{\left(19\ \frac{\text{L}}{\text{s}}\right)(125\ \text{Pa})}{10^6\ \frac{\text{L·W}}{\text{m}^3\text{·kW}}}$$
$$= 2.375\times10^{-3}\ \text{kW}$$

In order to predict the performance of a dynamically similar fan, use Eq. 41.28.

$$\frac{\text{AkW}_1}{\text{AkW}_2} = \left(\frac{D_1}{D_2}\right)^5\left(\frac{n_1}{n_2}\right)^3\left(\frac{\gamma_1}{\gamma_2}\right)$$
$$\text{AkW}_2 = \text{AkW}_1\left(\frac{D_2}{D_1}\right)^5\left(\frac{n_2}{n_1}\right)^3\left(\frac{\gamma_2}{\gamma_1}\right)$$
$$\frac{D_2}{D_1} = \frac{1}{\frac{1}{8}} = 8$$
$$\frac{n_2}{n_1} = \frac{1}{2}$$

At standard air conditions, $\rho = 1.2\ \text{kg/m}^3$. Use standard atmospheric data from App. 25.E for 1500 m altitude.

$$\rho_{1500\ \text{m}} = \rho_2 = \frac{p}{RT} = \frac{(0.8456\ \text{bar})\left(10^5\ \frac{\text{Pa}}{\text{bar}}\right)}{\left(287\ \frac{\text{J}}{\text{kg·K}}\right)(278.4\text{K})}$$
$$= 1.058\ \text{kg/m}^3$$

$$\frac{\gamma_2}{\gamma_1} = \frac{\rho_2 g}{\rho_1 g} = \frac{\rho_2}{\rho_1} = \frac{1.058\ \frac{\text{kg}}{\text{m}^3}}{1.2\ \frac{\text{kg}}{\text{m}^3}}$$
$$= 0.882$$

$$\text{AkW}_2 = (2.375\times10^{-3}\ \text{kW})(8)^5\left(\tfrac{1}{2}\right)^3(0.882)$$
$$= \boxed{8.58\ \text{kW}\quad(8.6\ \text{kW})}$$

The answer is (A).

HVAC

(e) The maximum stress in each bolt is most nearly

(A) 42,200 lbf/in^2 (2.900×10^8 Pa)

(B) 42,300 lbf/in^2 (2.905×10^8 Pa)

(C) 42,700 lbf/in^2 (2.930×10^8 Pa)

(D) 42,900 lbf/in^2 (2.940×10^8 Pa)

(f) The minimum stress in each bolt is most nearly

(A) 42,220 lbf/in^2 (2.899×10^8 Pa)

(B) 42,240 lbf/in^2 (2.900×10^8 Pa)

(C) 42,300 lbf/in^2 (2.904×10^8 Pa)

(D) 42,450 lbf/in^2 (2.915×10^8 Pa)

(g) The mean bolt stress is most nearly

(A) 42,220 lbf/in^2 (280 MPa)

(B) 42,300 lbf/in^2 (290 MPa)

(C) 42,500 lbf/in^2 (291 MPa)

(D) 42,700 lbf/in^2 (293 MPa)

(h) The ideal endurance strength of the bolt is most nearly

(A) 40,000 lbf/in^2 (280 MPa)

(B) 45,000 lbf/in^2 (310 MPa)

(C) 50,000 lbf/in^2 (345 MPa)

(D) 55,000 lbf/in^2 (380 MPa)

(i) Using a thread stress concentration factor of 2 and appropriate endurance strength derating factors for a 95% reliability, disregarding miscellaneous effects such as residual stresses and corrosion, the factor of safety is most nearly

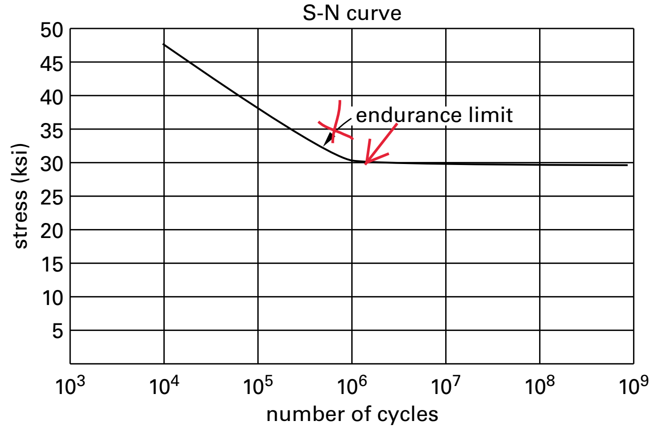
(A) 1.2

(B) 1.8

(C) 2.1

(D) 2.5

5. A structural member with the S-N curve shown is subjected to repeated loadings. 10% of the time, the member experiences cycles at 117% of the endurance strength; 15% of the time, the cycles are at 110% of the endurance strength; and 20% of the time, the cycles are at 105% of the endurance strength. The rest of the time, the stress is below the endurance limit. How many cycles can the member experience before failure?



(A) 390,000

(B) 450,000

(C) 620,000

(D) 730,000

Materials

The bending geometry factor, J, is determined from AGMA Standard 2001. For a spur gear having 24 teeth,

$$J = 0.25$$

The tooth strength, S_t, is

$$S_t = \left(100{,}000\ \frac{\text{lbf}}{\text{in}}\right)\left(\frac{P}{J}\right) = \left(100{,}000\ \frac{\text{lbf}}{\text{in}}\right)\left(\frac{1.6\ \text{in}^{-1}}{0.25}\right)$$
$$= \boxed{640{,}000\ \text{lbf/in}^2}$$

The answer is (D).

28. From the allowable contact stress, s_{ac}, tables in AGMA Standard 2001, the range of allowable contact stress is approximately 85,000 lbf/in^2 to 95,000 lbf/in^2. The largest allowable contact stress value, 95,000 lbf/in^2, is selected to calculate the maximum permissible calculated contact stress.

From AGMA Standard 2001, the pitting life factor, C_L, is 1.00 for 10,000,000 load cycles.

The maximum permissible calculated contact stress, s_c, is

$$s_c = s_{ac}\left(\frac{C_L C_H}{C_T C_R}\right) = \left(95{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{(1.00)(1.05)}{(1.00)(1.50)}\right)$$
$$= \boxed{66{,}500\ \text{lbf/in}^2 \quad (67{,}000\ \text{lbf/in}^2)}$$

The answer is (C).

29. To determine the runout tolerance from AGMA Standard 2000, the diametral pitch and pitch diameter must be calculated. The diametral pitch, P, is

$$P = \frac{\pi}{p} = \frac{\pi}{0.785\ \text{in}} = 4.00\ \text{in}^{-1}$$

The pitch diameter, d, is

$$d = \frac{N}{P} = \frac{24}{4.00\ \text{in}^{-1}} = 6.00\ \text{in}$$

From AGMA Standard 2000, the runout tolerance is $\boxed{0.00166\ \text{in}\ (0.0017\ \text{in.})}$

The answer is (C).

30. *Customary U.S. Solution*

From Eq. 54.103, the torque per contact surface is

$$T = \pi f p_{\max} r_i (r_o^2 - r_i^2)$$
$$= \pi(0.12)\left(100\ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{2.5\ \text{in}}{2}\right)\left(\left(\frac{4.5\ \text{in}}{2}\right)^2 - \left(\frac{2.5\ \text{in}}{2}\right)^2\right)$$
$$= 164.9\ \text{in-lbf}$$

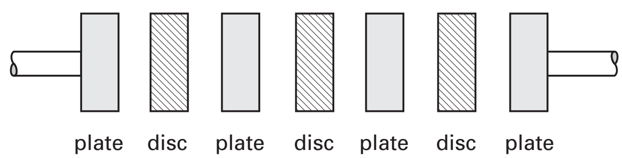
The slipping torque is

$$T_{\text{slip}} = 3T_{\text{rated}} = (3)(300\ \text{in-lbf}) = 900\ \text{in-lbf}$$

The slipping torque is equal to the torque per contact surface.

$$T_{\text{slip}} = T$$
$$900\ \text{in-lbf} = 164.9M \quad [\text{in in-lbf}]$$
$$M = 5.5$$

Use six contact surfaces. The arrangement is



(a) $\boxed{\text{Use four plates.}}$

The answer is (C).

(b) $\boxed{\text{Use three discs.}}$

The answer is (B).

SI Solution

Assume uniform wear. From Eq. 54.103, the torque per contact surface is

$$T = \pi f p_{\max} r_i (r_o^2 - r_i^2)$$
$$= \frac{\pi(0.12)(700\ \text{kPa})\left(1000\ \frac{\text{Pa}}{\text{kPa}}\right)\left(\frac{65\ \text{mm}}{2}\right) \times \left(\left(\frac{115\ \text{mm}}{2}\right)^2 - \left(\frac{65\ \text{mm}}{2}\right)^2\right)}{\left(1000\ \frac{\text{mm}}{\text{m}}\right)^3}$$
$$= 19.29\ \text{N·m}$$

32 Hz is not close to any frequency in commercial use. Try $p = 6$.

$$f = \frac{(6)\left(960\ \frac{\text{rev}}{\text{min}}\right)}{(2)\left(60\ \frac{\text{sec}}{\text{min}}\right)(1-0)} = 48\ \text{Hz}$$

With a 4% slip, $f = 50$ Hz.

$$\boxed{50\ \text{Hz (European)}}$$

The answer is (D).

8. A synchronous motor's speed is dependent on the speed of the stator rotating field. The motor can rotate at the speed of the applied alternating current or submultiples thereof. From Eq. 72.76,

$$n_{\text{synchronous}} = \frac{120f}{p}$$

The synchronous speed is 120 times the frequency of the electric source divided by the number of poles.

The answer is (A).

9. The total real power is

$$\begin{aligned} P_t &= \frac{Q_t}{\tan\phi} \\ &= \frac{(45\ \text{MVAR})\left(1000\ \frac{\text{kVAR}}{\text{MVAR}}\right)}{\tan 25^\circ} \\ &= 96{,}502\ \text{kVAR} \end{aligned}$$

From Eq. 72.67, the line current is

$$\begin{aligned} I_l &= \frac{P_t}{\sqrt{3}\,V_l\cos\phi} \\ &= \frac{96{,}502\ \text{kVAR}}{(\sqrt{3})(22\ \text{kV})\left(1000\ \frac{\text{A}}{\text{kA}}\right)\cos 25^\circ} \\ &= \boxed{2.79\ \text{kA}\quad(2.8\ \text{kA})} \end{aligned}$$

The answer is (C).

10. (a) Use Eq. ~~36.6~~ 42.6(c) to solve for the temperature leaving the air handler. The constant 1.08 is the product of an air density of 0.075 lbm/ft^3, a specific heat of 0.24 Btu/lbm-°F, and a conversion of 60 min/hr.

$$\begin{aligned} T_{\text{out}} &= \frac{\dot{q}}{\left(1.08\ \frac{\text{Btu-min}}{\text{ft}^3\text{-hr-°F}}\right)\dot{V}} + T_{\text{in}} \\ &= \frac{(40\ \text{kW})\left(1000\ \frac{\text{W}}{\text{kW}}\right)\left(3.412\ \frac{\text{Btu}}{\text{W-hr}}\right)}{\left(1.08\ \frac{\text{Btu-min}}{\text{ft}^3\text{-hr-°F}}\right)\left(5000\ \frac{\text{ft}^3}{\text{min}}\right)} + 60°\text{F} \\ &= \boxed{85.27°\text{F}\quad(85°\text{F})} \end{aligned}$$

The answer is (D).

(b) Since the load is purely resistive, the power factor is 1.0. The load is three-phase. From Eq. 72.68, the line current is

$$I_l = \frac{P}{\sqrt{3}\,V_l} = \frac{(40\ \text{kW})\left(1000\ \frac{\text{W}}{\text{kW}}\right)}{(\sqrt{3})(460\ \text{V})} = \boxed{50.2\ \text{A}\quad(50\ \text{A})}$$

The answer is (B).

11. From Eq. 72.72, the total real electrical power drawn from the line by the motor is

$$\begin{aligned} P_{\text{electrical}} &= \frac{P_{\text{rated}}}{\eta_m} = \frac{(25\ \text{hp})\left(745.7\ \frac{\text{W}}{\text{hp}}\right)}{(0.92)\left(1000\ \frac{\text{W}}{\text{kW}}\right)} \\ &= 20.26\ \text{kW} \end{aligned}$$

Rearrange Eq. 72.79 for the power factor.

$$\begin{aligned} \text{pf} &= \frac{P_{\text{electrical}}}{\sqrt{3}\,V_l I_l} = \frac{(20.26\ \text{kW})\left(1000\ \frac{\text{W}}{\text{kW}}\right)}{(\sqrt{3})(480\ \text{V})(28\ \text{A})} \\ &= 0.870 \end{aligned}$$

The power factor angle is

$$\begin{aligned} \phi &= \arccos \text{pf} = \arccos 0.870 \\ &= 29.5° \end{aligned}$$