For most cast-in-place floor systems, the slab and beams are cast monolithically and the slab functions as the flange of a T- or L-shaped beam, as shown in Fig. 3.3. ACI Sec. 6.3.2 limits the effective flange width, b_e, of such members by the following criteria.

Slab Extending Both Sides (T-Beam)

l-sub-n/4 + b-sub-w

$$b_{e,\text{int}} \leq \begin{cases} l_n/4 \\ b_w + 16h_s \\ b_w + \dfrac{s_1 + s_2}{2} \end{cases} \qquad 3.9$$

Slab Extending One Side Only (L-Beam)

$$b_{e,\text{ext}} \leq \begin{cases} b_w + \dfrac{l_n}{12} \\ b_w + 6h_s \\ b_w + \dfrac{s_1}{2} \end{cases} \qquad 3.10$$

l_n is the length of the clear span measured from face to face of beams or other supports. Other symbols are as defined in Fig. 3.3.

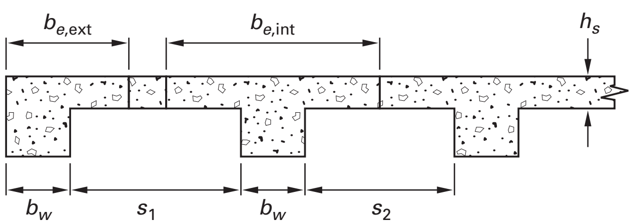
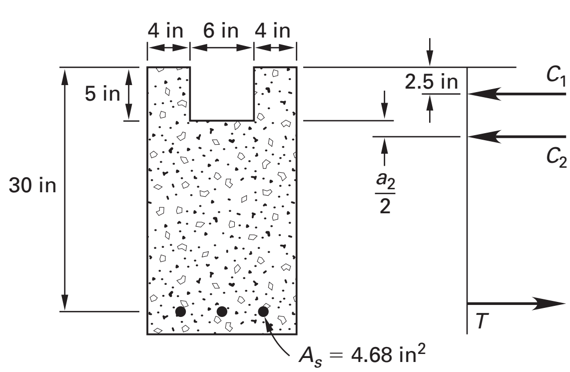


Figure 3.3 Effective Widths of T-Beams and L-Beams

Example 3.2
Analysis of an Irregularly Shaped Beam

Calculate the design moment strength of the section shown. The compressive strength of the concrete is 4000 psi, and the yield stress of the reinforcement is 60,000 psi.



Solution:

The equivalent area of the compression zone can be found from Eq. 3.8.

$$\begin{aligned} A_c &= \frac{A_s f_y}{0.85 f'_c} \\ &= \frac{(4.68\ \text{in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)} \\ &= 82.6\ \text{in}^2 \end{aligned}$$

Because the equivalent area of the compression zone exceeds the areas in the rectangular regions to the left and right of the trough, the compression zone extends to some depth below the bottom of the trough. This depth is

$$\begin{aligned} a_2 &= \frac{A_c - 2b_1h_1}{b_w} \\ &= \frac{82.6\ \text{in}^2 - (2)(4\ \text{in})(5\ \text{in})}{14\ \text{in}} \\ &= 3.04\ \text{in} \end{aligned}$$

The equivalent compression force can be expressed in terms of a component acting in the rectangular regions adjacent to the trough, C_1, and a component acting over the region below the trough, C_2.

$$\begin{aligned} C_1 &= 2(0.85 f'_c b_1 h_1) \\ &= (2)(0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(4\ \text{in})(5\ \text{in}) \\ &= 136\ \text{kip} \\ C_2 &= 0.85 f'_c b_w a_2 \\ &= (0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(14\ \text{in})(3.04\ \text{in}) \\ &= 145\ \text{kip} \end{aligned}$$

Taking moments of the two forces about the line of action of the tension force gives the design moment strength of the section.

$$\begin{aligned} \phi M_n &= \phi\left(C_1\left(d - \frac{h_1}{2}\right) + C_2\left(d - h_1 - \frac{a_2}{2}\right)\right) \\ &= (0.9)\left(\begin{array}{l}(136\ \text{kip})\left(30\ \text{in} - \dfrac{5\ \text{in}}{2}\right) \\ +\ (145\ \text{kip})\left(30\ \text{in} - 5\ \text{in} - \dfrac{3.04\ \text{in}}{2}\right)\end{array}\right) \\ &= (6430\ \text{in-kip})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) \\ &= 536\ \text{ft-kip} \end{aligned}$$

If the axial load on the column is greater than $0.3f'_c A_g$ or f'_c is greater than 10,000 psi, an additional requirement is imposed by ACI Sec. 18.7.5.4.

$$A_{sh} \geq 0.2k_f k_n \frac{P_u s b_c}{f_{yt} A_{ch}}$$

$$k_n = \frac{n_l}{n_l - 2}$$

- b_c is the dimension of the hoop confining the concrete core, ~~measured from center to center~~. Spacing of the transverse reinforcement must not exceed

$$s \leq \begin{cases} 0.25 \times \text{least column dimension} \\ 6d_b \quad \text{[of longitudinal steel]} \\ s_o \end{cases} \qquad \textbf{11.6}$$

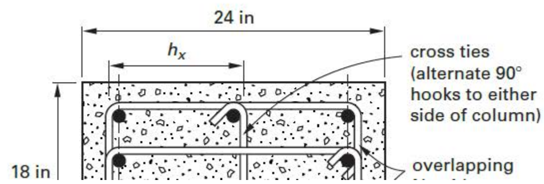
s_o is calculated from ACI Eq. 18.7.5.3, but may not be taken as less than 4 in or more than 6 in.

$$s_o = 4 + \frac{14 - h_x}{3} \quad [4 \text{ in} \leq s_o \leq 6 \text{ in}] \qquad \textbf{11.7}$$

h_x is the maximum horizontal spacing of hoop

Example 11.2
Transverse Reinforcement for a Rectangular Column in a Special Moment Frame

The rectangular reinforced concrete column shown is part of a special moment frame in a region of high seismic risk. The controlling value of P_u is less than $0.3f'_c A_g$. Concrete is normal weight with specified compressive strength of 5000 psi and steel has a specified yield stress of 60,000 psi. 10 no. 9 bars reinforce the cross section longitudinally in the pattern shown. Determine the required spacing of no. 4 hoops and cross ties in the vicinity of the joint face, and the extent of the hoops at that spacing for an unsupported column length of 10 ft.



measured to outside edges of transverse reinforcement.

For the strong axis direction, with four no. 4 legs of the hoops confining the concrete,

$$b_c = 24 \text{ in} - 3 \text{ in} - \sout{0.5 \text{ in}}$$
$$= \sout{20.5 \text{ in}}\ \text{21.0 in}$$
$$s \le \frac{A_{sh}}{0.0093 b_c} = \frac{(4)(0.2 \text{ in}^2)}{(0.0093)(\sout{20.5 \text{ in}})}\ \text{21.0 in}$$
$$= \sout{4.2 \text{ in}}\ \text{4.1 in}$$

In the weak direction, with ~~two~~ three legs furnishing confinement,

$$b_c = 18 \text{ in} - 3 \text{ in} - \sout{0.5 \text{ in}}$$
$$= \sout{14.5 \text{ in}}\ \text{15.0 in}$$
$$s \le \frac{A_{sh}}{0.0093 b_c} = \frac{(3)(0.2 \text{ in}^2)}{(0.0093)(\sout{14.5 \text{ in}})}\ \text{15.0 in}$$
$$= \sout{4.4 \text{ in}}\ \text{4.3 in}$$

Other spacing limits of ACI Sec. 18.7.5.2 require

$$\begin{aligned} h_x &= \frac{h}{2} - \text{cover} - d_h + \frac{d_{bl}}{2} \\ &= 12 \text{ in} - 1.5 \text{ in} - 0.5 \text{ in} + \frac{1.125 \text{ in}}{2} \\ &= 10.6 \text{ in} \end{aligned}$$

From Eq. 11.7,

$$\begin{aligned} s_o &= 4 \text{ in} + \left(\frac{14 \text{ in} - h_x}{3}\right) \\ &= 4 \text{ in} + \left(\frac{14 \text{ in} - 10.6 \text{ in}}{3}\right) \\ &= 5.1 \text{ in} \end{aligned}$$

From Eq. 11.6,

$$s \le \begin{cases} 0.25 \times \text{least column dimension} \\ \quad = (0.25)(18 \text{ in}) \\ \quad = 4.5 \text{ in} \quad \text{[controls]} \\ 6d_b = (6)(1.125 \text{ in}) \quad \text{[of longitudinal steel]} \\ \quad = 6.75 \text{ in} \\ s_o = 5.1 \text{ in} \end{cases}$$

The spacing of ~~4.2~~ 4.1 in computed for the strong axis confinement controls (say, 4 in on centers). From Eq. 11.8, hoops are required at a distance from the face of the joint of

$$l_o \ge \begin{cases} \text{column depth at joint face} \\ \quad = 24 \text{ in} \quad \text{[controls]} \\ \frac{1}{6} \times \text{column clear span} \\ \quad = \left(\frac{1}{6}\right)(10 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right) \\ \quad = 20 \text{ in} \\ 18 \text{ in} \end{cases}$$

3. Joints in Special Moment Frame Members

ACI Sec. 18.8 gives requirements for the design of beam-column joints in special moment frames. Forces in reinforcement allow for overstrength using a tensile stress of $1.25 f_y$. Because forces are conservatively estimated, ACI Sec. 21.2.4.3 specifies the capacity reduction factor, ϕ, to be 0.85 for shear in the joints rather than the traditional 0.75 for ordinary shear calculations.

ACI Secs. 18.8.2 and 18.8.5 specify the following requirements for longitudinal bar development (no. 3 through no. 11 bars).

- Bars that terminate must extend to the far face of the confined concrete in the core.
- For bars that continue through the joint, the width of joint parallel to the reinforcement must be greater than or equal to $20d_b$ of the largest longitudinal bar for normal weight concrete, and at least $26d_b$ for lightweight concrete.
- For 90° hooks in tension,

$$l_{dh} \ge \begin{cases} \dfrac{f_y d_b}{65\lambda\sqrt{f'_c}} \\ 6d_b/\lambda \\ (6 \text{ in})/\lambda \end{cases} \qquad 11.9$$

- For straight embedment in tension, l_d is 2.5 times the development length computed for a 90° hook if the depth of fresh concrete cast beneath the bar is less than 12 in, and is 3.25 times the 90° hook development length if the depth is 12 in or more.
- If any portion of the straight embedment length is outside the confined core, the required development length is

$$l_{dm} = 1.6 l_d - 0.6 l_{dcc} \qquad 11.10$$

  l_{dm} is the required development length when not fully within the core, and l_{dcc} is the length embedded in confined concrete.
- Development length for bars in compression is the same as for ordinary bars not subjected to stress reversals.

When there is confinement from framing members on four sides, and the width of every framing member is at least three-fourths the column width, the following provisions apply.

The required area of positive steel at the critical location is 6.32 in^2. The distance from support D to the point where 50% of the bottom steel can terminate on the left side is most nearly

(A) 2.8 ft
(B) 6.4 ft
(C) 9.2 ft
(D) 12 ft

Design Criteria

- $f'_c = 4$ ksi, hard rock aggregate
- $f_y = 60$ ksi for principal reinforcement

Solution

The effective width of the T-beam for the positive region is

$$b_e \le \begin{cases} \frac{L}{4} = \frac{(36\text{ ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{4} \\ \quad = 108\text{ in} \quad \text{[controls]} \\ b_w + 16h = 18\text{ in} + (16)(8\text{ in}) \\ \quad = 146\text{ in} \\ s_l + s_r = \left(\frac{22\text{ ft}}{2} + \frac{22\text{ ft}}{2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right) \\ \quad = 264\text{ in} \end{cases}$$

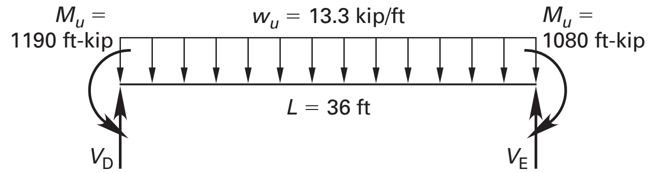
For the remaining 50% of the bottom steel,

$$\begin{aligned} A_s &= (0.5)(6.32\text{ in}^2) \\ &= 3.16\text{ in}^2 \end{aligned}$$

$$\begin{aligned} a &= \frac{A_s f_y}{0.85 f'_c b_e} = \frac{(3.16\text{ in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)}{(0.85)\left(4\ \frac{\text{kip}}{\text{in}^2}\right)(108\text{ in})} \\ &= 0.52\text{ in} \\ &< h = 8\text{ in} \end{aligned}$$

$$\begin{aligned} c &= \frac{a}{\beta_1} = \frac{0.52\text{ in}}{0.85} \\ &= 0.61\text{ in} \\ &< \frac{3d}{8} \quad [\text{tension controlled, } \phi = 0.9] \end{aligned}$$

$$\begin{aligned} \phi M_n &= \phi A_s f_y \left(d - \frac{a}{2}\right) \\ &= \frac{(0.9)(3.16\text{ in}^2)\left(60\ \frac{\text{kip}}{\text{in}^2}\right)\left(33.5\text{ in} - \frac{0.52\text{ in}}{2}\right)}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 473\text{ kip-ft} \end{aligned}$$



$$\begin{aligned} \sum M_E &= 0\text{ ft-kip} \\ &= V_D L - \frac{w_u L^2}{2} - M_{u,D} - M_{u,E} \\ V_D &= \frac{w_u L}{2} + \frac{M_{u,D}}{L} + \frac{M_{u,E}}{L} \\ &= \frac{\left(13.3\ \frac{\text{kip}}{\text{ft}}\right)(36\text{ ft})}{2} + \frac{1190\text{ ft-kip}}{36\text{ ft}} - \frac{1080\text{ ft-kip}}{36\text{ ft}} \\ &= 242.5\text{ kip} \end{aligned}$$

The moment at a distance x from D is

$$\begin{aligned} M_u &= M_{u,D} + V_D x - \frac{w_u x^2}{2} \\ &= -1190\text{ ft-kip} + (242.5\text{ kip})x - \frac{\left(13.3\ \frac{\text{kip}}{\text{ft}}\right)x^2}{2} \end{aligned}$$

Equating the moment equation to the design moment and solving for x gives the distance to the theoretical cutoff point for 50% of the bottom steel.

$$\begin{aligned} \phi M_n &= M_u \\ 473\text{ ft-kip} &= -1190\text{ ft-kip} + (242.5\text{ kip})x - \left(13.3\ \frac{\text{kip}}{\text{ft}}\right)\frac{x^2}{2} \\ x &= 9.16\text{ ft} \end{aligned}$$

ACI requires that bars must extend beyond the point where they are theoretically no longer needed to a distance of

$$\text{extension} \ge \begin{cases} 12d_b = (12)(1\text{ in}) = 12\text{ in} \\ d = 33.5\text{ in} \quad \text{[controls]} \end{cases}$$

Therefore, the required cutoff point is

$$\begin{aligned} x' &= 9.16\text{ ft} - \frac{33.5\text{ in}}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 6.37\text{ ft} \quad (6.4\text{ ft}) \end{aligned}$$

The answer is (B).