angle	value	number of measurements
1	63°	2
2	77°	6
3	41°	5

Solution

The total of the angles is

$$63° + 77° + 41° = 181°$$

So, $-1°$ must be divided among the three angles. These corrections are inversely proportional to the number of measurements. The sum of the measurement inverses is

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{5} = 0.5 + 0.167 + 0.2 = 0.867$$

The most probable value of angle 1 is then

$$63° + \frac{\left(\frac{1}{2}\right)(-1°)}{0.867} = 62.42°$$

Example 78.8

The interior angles of a triangular traverse were measured. What is the most probable value of angle 1?

angle	value
1	$63° \pm 0.01°$
2	$77° \pm 0.03°$
3	$41° \pm 0.02°$

Solution

The total of the angles is

$$63° + 77° + 41° = 181°$$

So, $-1°$ must be divided among the three angles. The corrections are proportional to the square of the probable errors.

$$(0.01)^2 + (0.03)^2 + (0.02)^2 = 0.0014$$

The most probable value of angle 1 is

$$63° + \frac{(0.01)^2(-1°)}{0.0014} = 62.93°$$

3. ERRORS IN COMPUTED QUANTITIES

When independent quantities with known errors are added or subtracted, the error of the result is given by Eq. 78.6. The squared terms under the radical are added regardless of whether the calculation is addition or subtraction.

$$E_{\text{sum}} = \sqrt{E_1^2 + E_2^2 + E_3^2 + \cdots} \qquad 78.6$$

The error in the product of two quantities, x_1 and x_2, which have known errors, E_1 and E_2, is given by Eq. 78.7.

$$E_{\text{product}} = \sqrt{x_1^2 E_2^2 + x_2^2 E_1^2} \qquad 78.7$$

Example 78.9

An electronic distance measurement (EDM) instrument manufacturer has indicated that the manufacturer's standard error with a particular instrument is ± 0.04 ft $\pm$ 10 ppm. The instrument is used to measure a distance of 3000 ft. (a) What is the precision of the measurement? (b) What is the expected error of the measurement in ppm?

Solution

(a) There are two independent errors here. The fixed error is ± 0.04 ft. The variable error is

$$E_{\text{variable}} = (3000 \text{ ft})\left(\frac{10}{1{,}000{,}000}\right) = 0.03 \text{ ft}$$

The precision is $\pm(0.04 \text{ ft} + 0.03 \text{ ft}) = \pm 0.07$ ft.

(b) The expected error is

23.3 ppm

$$E_{\text{expected}} = \frac{0.07 \text{ ft}}{3000 \text{ ft}} \times 10^6 = 233 \text{ ppm}$$

Example 78.10

The sides of a rectangular section were determined to be 1204.77 ft $\pm$ 0.09 ft and 765.31 ft $\pm$ 0.04 ft, respectively. What is the probable error in area?

Solution

Use Eq. 78.7.

$$\begin{aligned} E_{\text{area}} &= \sqrt{x_1^2 E_2^2 + x_2^2 E_1^2} \\ &= \sqrt{(1204.77 \text{ ft})^2 (0.04 \text{ ft})^2 + (765.31 \text{ ft})^2 (0.09 \text{ ft})^2} \\ &= 84.06 \text{ ft}^2 \end{aligned}$$

Transportation