Description

Equation 10.51 through Eq. 10.62 give the area, centroids, and moments of inertia for rectangles.

Example

A 12 cm wide × 8 cm high rectangle is placed such that its centroid is located at the origin, (0, 0). What is the percentage change in the product of inertia if the rectangle is rotated 90° counterclockwise about the origin?

(A) –32% (decrease)

(B) 0%

(C) 32% (increase)

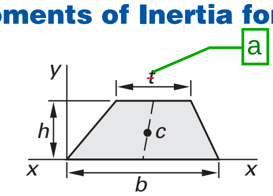
(D) 64% (increase)

Solution

The product of inertia is zero whenever one or more of the reference axes are lines of symmetry. In this case, both axes are lines of symmetry before and after the rotation. From Eq. 10.61, $I_{x_c y_c} = 0$.

The answer is (B).

Equation 10.63 Through Eq. 10.68: Centroid and Area Moments of Inertia for Trapezoids



area and centroid

$$A = h(a+b)/2 \qquad 10.63$$

$$y_c = \frac{h(2a+b)}{3(a+b)} \qquad 10.64$$

area moment of inertia

$$I_{x_c} = \frac{h^3(a^2+4ab+b^2)}{36(a+b)} \qquad 10.65$$

$$I_x = \frac{h^3(3a+b)}{12} \qquad 10.66$$

$(radius\ of\ gyration)^2$

$$r_{x_c}^2 = \frac{h^2(a^2+4ab+b^2)}{18(a+b)} \qquad 10.67$$

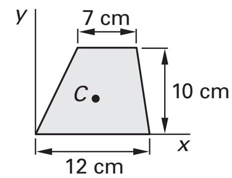
$$r_x^2 = \frac{h^2(3a+b)}{6(a+b)} \qquad 10.68$$

Description

Equation 10.63 through Eq. 10.68 give the area, centroids, and moments of inertia for trapezoids.

Example

What are most nearly the area and the y-coordinate, respectively, of the centroid of the trapezoid shown?



(A) 95 cm^2; 4.6 cm

(B) 110 cm^2; 5.4 cm

(C) 120 cm^2; 6.1 cm

(D) 140 cm^2; 7.2 cm

Solution

The area of the trapezoid is

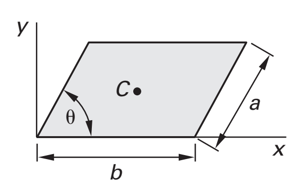
$$A = h(a+b)/2 = \frac{(10\text{ cm})(7\text{ cm} + 12\text{ cm})}{2} = 95\text{ cm}^2$$

From Eq. 10.64, the y-coordinate of the centroid of the trapezoid is

$$y_c = \frac{h(2a+b)}{3(a+b)} = \frac{(10\text{ cm})\big((2)(7\text{ cm}) + 12\text{ cm}\big)}{(3)(7\text{ cm} + 12\text{ cm})} = 4.56\text{ cm} \quad (4.6\text{ cm})$$

The answer is (A).

Equation 10.69 Through Eq. 10.80: Centroid and Area Moments of Inertia for Rhomboids



area and centroid

$$A = ab\sin\theta \qquad 10.69$$

$$x_c = (b + a\cos\theta)/2 \qquad 10.70$$

$$y_c = (a\sin\theta)/2 \qquad 10.71$$