With *critical damping*, the damping ratio is equal to 1. There is no overshoot, and the behavior reaches the steady-state equilibrium condition the fastest of the three cases, without oscillations. The characteristic equation of critically damped systems has two identical real roots (zeros).

Equation 4.9 Through Eq. 4.14: Roots of the Characteristic Equation

$$r_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2} \qquad 4.9$$

$$y = C_1 e^{r_1 x} + C_2 e^{r_2 x} \qquad 4.10$$

$$y = (C_1 + C_2 x) e^{r_1 x} \qquad 4.11$$

$$y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x) \qquad 4.12$$

$$\alpha = -a/2 \qquad 4.13$$

$$\beta = \frac{\sqrt{4b - a^2}}{2} \qquad 4.14$$

Description

The roots of the characteristic equation are given by the quadratic equation, Eq. 4.9.

If $a^2 > 4b$, then the two roots are real and different, and the solution is overdamped, as shown in Eq. 4.10.

If $a^2 = 4b$, then the two roots are real and the same (i.e., are *double roots*), and the solution is critically damped, as shown in Eq. 4.11.

If $a^2 < 4b$, then the two roots are imaginary and of the form $(\alpha + i\beta)$ and $(\alpha - i\beta)$, and the solution is underdamped, as shown in Eq. 4.12.

Example

What is the general solution to the following homogeneous differential equation?

$$y'' - 8y' + 16y = 0$$

(A) $y = C_1 e^{4x}$

(B) $y = (C_1 + C_2 x) e^{4x}$

(C) $y = C_1 e^{-4x} + C_2 e^{4x}$

(D) $y = C_1 e^{2x} + C_2 e^{4x}$

Solution

Find the roots of the characteristic equation.

$$r^2 - 8r + 16 = 0$$

$$a = -8$$

$$b = 16$$

From Eq. 4.9,

$$\begin{aligned} r_{1,2} &= \frac{-a \pm \sqrt{a^2 - 4b}}{2} \\ &= \frac{-(-8) \pm 2\sqrt{(-8)^2 - (4)(16)}}{2} \\ &= 4, 4 \end{aligned}$$

Because $a^2 = 4b$, the characteristic equation has double roots. With $r = 4$, the solution takes the form

$$\begin{aligned} y &= (C_1 + C_2 x) e^{rx} \\ &= (C_1 + C_2 x) e^{4x} \end{aligned}$$

The answer is (B).

3. LINEAR NONHOMOGENEOUS DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

In a nonhomogeneous differential equation, the sum of derivative terms is equal to a nonzero *forcing function* of the independent variable (i.e., $f(x)$ in Eq. 4.1 is nonzero). In order to solve a nonhomogeneous equation, it is often necessary to solve the homogeneous equation first. The homogeneous equation corresponding to a nonhomogeneous equation is known as the *reduced equation* or *complementary equation*.

Equation 4.15: Complete Solution to Nonhomogeneous Differential Equation

$$y(x) = y_h(x) + y_p(x) \qquad 4.15$$

Description

The complete solution to the nonhomogeneous differential equation is shown in Eq. 4.15. The term $y_h(x)$ is the *complementary solution*, which solves the complementary (i.e., homogeneous) case. The *particular solution*, $y_p(x)$, is any specific solution to the nonhomogeneous Eq. 4.1 that is known or can be found. Initial values are used to evaluate any unknown coefficients in the complementary solution after $y_h(x)$ and $y_p(x)$ have been combined. The particular solution will not have any unknown coefficients.

Table 4.1: Method of Undetermined Coefficients

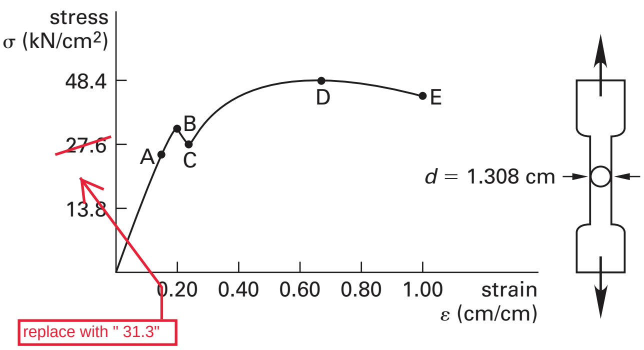
Table 4.1 *Method of Undetermined Coefficients*

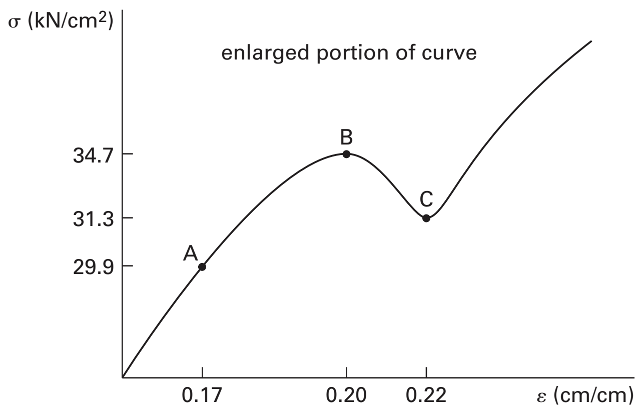
form of $f(x)$	form of $y_p(x)$
A	B
$Ae^{\alpha x}$	$Be^{\alpha x},\ a \neq r_n$
$A_1 \sin \omega x + A_2 \cos \omega x$	$B_1 \sin \omega x + B_2 \cos \omega x$

Diagnostic Exam

Topic VII: Material Properties and Processing

1. The results of a tensile test on a round specimen of a given material are shown.





What is most nearly the yield stress?

(A) 14 kN/cm^2

(B) 26 kN/cm^2

(C) 31 kN/cm^2

(D) 48 kN/cm^2

2. The activation energy for creep is 161 kJ/mol for a given alloy. If the applied stress is fixed and the stress sensitivity remains the same, by approximately what factor does the creep rate change when the temperature increases from 350°C to 450°C?

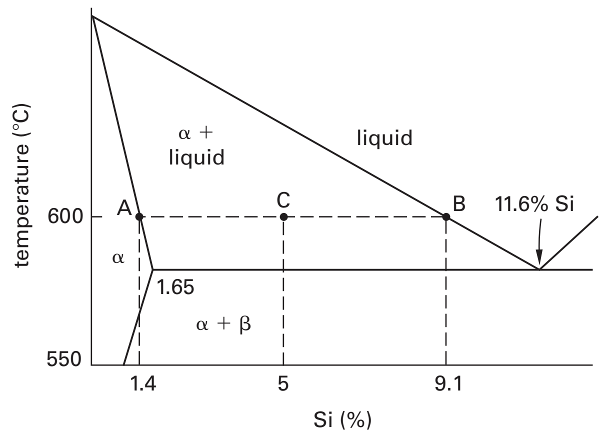
(A) 2.2

(B) 3.0

(C) 74

(D) 220

3. Using the phase diagram given, what is most nearly the percentage of liquid remaining at 600°C that results from the equilibrium cooling of an alloy containing 5% silicon and 95% aluminum?



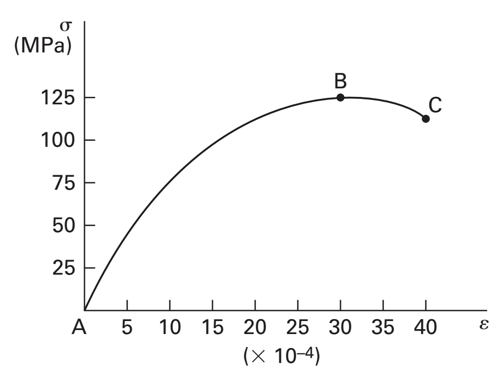
(A) 0.0%

(B) 47%

(C) 53%

(D) 67%

4. The stress-strain curve for a nonlinear, perfectly elastic material is shown. A sample of the material is loaded until the stress reaches the value at point B. Then, the material is unloaded to zero stress.



What is most nearly the permanent set in the material?

(A) 0

(B) 0.001

(C) 0.002

(D) 0.003

friction can be calculated from Eq. 24.7 (or the variation equation, depending on the travel direction) using $\phi = 0$.

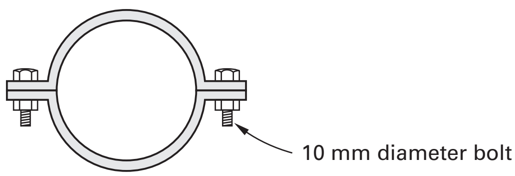
$$\eta_m = \frac{M_{f=0}}{M}$$

In the absence of an antifriction ring, an additional torque will be required to overcome friction in the collar. Since the collar is generally flat, the normal force is the jack load for the purpose of calculating the frictional force.

$$M_{\text{collar}} = N\mu_{\text{collar}} r_{\text{collar}}$$

Example

The nuts on a collar are each tightened to 18 N·m torque. 17% of this torque is used to ~~overcome screw thread friction~~. The bolts have a nominal diameter of 10 mm. The threads are a simple square cut with a pitch angle of 15°. The coefficient of friction in the threads is 0.10.



replace with " tension the bolt"

What is the approximate tensile force in each bolt?

(A) 130 N

(B) 200 N

(C) 410 N

(D) 1600 N

Solution

The friction angle, ϕ, is

replace with " only 17% of the applied torque..."

$$\phi = \arctan \mu = \arctan 0.10 = 5.71°$$

Use Eq. 24.7. ~~Only the screw thread friction (17% of the total torque in this application)~~ contributes to the tensile force in the bolt. The force in the bolt is

$$P = \frac{M}{r\tan(\alpha+\phi)} = \frac{(0.17)(18\ \text{N·m})}{\left(\frac{0.01\ \text{m}}{2}\right)\tan(15° + 5.71°)} = 1619\ \text{N} \quad (1600\ \text{N})$$

The answer is (D).