53 Advanced Machine Design

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MACHINE DESIGN AND MATERIALS EXAM

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1. SPRINGS

An *ideal spring* is assumed to be perfectly elastic within its working range. The deflection is assumed to follow Hooke's law.[1] The force in a linear elastic spring can be found from Eq. 53.1.

Spring Energy

$$F_s = kx \qquad 53.1$$

[1]A spring can be perfectly elastic even though it does not follow Hooke's law. The deviation from proportionality, if any, occurs at very high loads. The difference in theoretical and actual spring forces is known as the *straight line error*.

The *spring constant*, k, is also known as the *stiffness*, *spring rate*, *scale*, and *k-value*.[2] The spring constant can be calculated as the ratio of the difference in forces applied to a spring over the difference in deflection of the spring.

$$k = \frac{F_1 - F_2}{x_1 - x_2} \qquad 53.2$$

A spring stores energy when it is compressed or extended. By the *work-energy principle*, the energy storage is equal to the work required to displace the spring. The potential energy of a spring whose ends have been displaced a total distance x is

Spring Energy

$$U = k\frac{x^2}{2} \qquad 53.3$$

If a mass, m, is dropped from a height h onto and is captured by a spring, the compression, x, can be found by equating the change in potential energy to the energy storage. Potential energy is equal to mgh, which can also be written as $mg(h + x)$. [Potential Energy]

$$mg(h + x) = \frac{1}{2}kx^2 \quad \text{[SI]} \qquad 53.4(a)$$

$$m\left(\frac{g}{g_c}\right)(h + x) = \frac{1}{2}kx^2 \quad \text{[U.S.]} \qquad 53.4(b)$$

Within the elastic region, this energy can be recovered by restoring the spring to its original unstressed condition. It is assumed that there is no permanent set, and no energy is lost through external friction or *hysteresis* (internal friction) when the spring returns to its original length.[3]

The entire applied load is felt by each spring in a series of springs linked end-to-end. The *equivalent* (*composite*) *spring constant* for springs in series is

$$\frac{1}{k_{\text{eq}}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots \quad \begin{bmatrix}\text{series}\\ \text{springs}\end{bmatrix} \qquad 53.5$$

Springs in parallel (e.g., concentric springs) share the applied load. The equivalent spring constant for springs in parallel is

$$k_{\text{eq}} = k_1 + k_2 + k_3 + \cdots \quad \begin{bmatrix}\text{parallel}\\ \text{springs}\end{bmatrix} \qquad 53.6$$

2. SPRING MATERIALS

A wide variety of materials are used for springs, including high-carbon steel, stainless steel and various alloys, nickel-based alloys (e.g., inconel), and copper-based alloys (e.g., phosphor-bronze and silicon-bronze). (See Table 53.1.) "Super-alloys" are used for high-temperature and highly corrosive environments. Spring rate, fatigue strength, temperature range, corrosion resistance, magnetic properties, and cost are all considerations.

Springs manufactured from prehardened materials are generally stress-relieved in a low-temperature process by heating to between 400°F and 800°F (200°C and 430°C) after forming. Springs with intricate shapes must be manufactured from annealed materials and be subsequently strengthened in high-temperature processes. They are first quenched to full hardness and then tempered. Age-hardenable materials (e.g., beryllium copper) can be strengthened simply by heating after forming.

Most springs are cold-wound. Springs with wire diameters much in excess of 1/2 in or 5/8 in (12 mm or 16 mm) are wound while red hot. Although the design methods are essentially the same for hot-wound and cold-wound springs, the allowable stresses are reduced approximately 20%, and the modulus is reduced slightly (approximately 9% for the shear modulus and approximately 5% for the elastic modulus). There are other unique issues and special needs associated with the manufacturing of hot-wound springs, as well.

Materials suitable for fatigue service include music wire (ASTM A228), carbon and alloy valve spring wire (ASTM A230), chrome-vanadium (ASTM A232), beryllium copper (ASTM B197), phosphor bronze (ASTM B159), and, to a lesser degree, type-302 stainless steel (ASTM A313). *Shotpeening* (*stresspeening*) is one of the best methods for increasing a spring's fatigue life.

3. ALLOWABLE SPRING STRESSES: STATIC LOADING

Helical compression and extension springs experience torsional shear stresses. The yield strength in shear is the theoretical maximum stress. There are three common ways of choosing the maximum allowable shear stress for static service.[4]

1. Selecting the allowable stress based on some percentage of the ultimate tensile strength is the most common method. For ferrous materials except for austenitic stainless, the percentage is approximately 45% to 65%. For nonferrous and austenitic stainless, the percentage is

[2]Another unfortunate name for the spring constant, k, that is occasionally encountered is the *spring index*. This is not the same as the spring index, C, used in helical coil spring design. The units will determine which meaning is intended.
[3]There is essentially no hysteresis in properly formed compression, extension, or open-wound helical torsion springs.
[4]These methods apply to helical compression and extension springs. Recommended percentages are different for other types of springs.

***Table 53.1** Properties of Typical Spring Materials (room temperature[a])*

material	modulus of elasticity (lbf/in^2)	shear modulus (lbf/in^2)	density (lbm/in^3)
high-carbon wire			
music wire ASTM A228	30×10^6	11.5×10^6	0.284
hard-drawn ASTM A227	30×10^6	11.5×10^6	0.284
oil-tempered ASTM A229	30×10^6	11.5×10^6	0.284
valve spring ASTM A230	30×10^6	11.5×10^6	0.284
alloy-steel wire			
chrome-vanadium SAE 6150, AISI 6150, ASTM A232	30×10^6	11.5×10^6	0.284
chrome-silicon AISI 9254, ASTM A401	30×10^6	11.5×10^6	0.284
silicon manganese AISI 9260	30×10^6	11.5×10^6	0.284
stainless steel wire			
AISI 302, ASTM A313	28×10^6	10.0×10^6	0.280
AISI 410, 420	28×10^6	11.0×10^6	0.286
17-7 PH[b]	29.5×10^6	11.0×10^6	0.286
18-2	29×10^6	9.8×10^6	0.272
nickel-chrome A286	29×10^6	10.4×10^6	0.290
copper alloys			
phosphor bronze ASTM B159	15×10^6	6.3×10^6	0.320
silicon bronze ASTM B99(A)	15×10^6	5.6×10^6	0.308
silicon bronze ASTM B99(A)	17×10^6	6.4×10^6	0.316
beryllium-copper ASTM B197	18.5×10^6	7.0×10^6	0.297
nickel alloys			
inconel 600	31×10^6	11.0×10^6	0.307
inconel X750	31.5×10^6	11.5×10^6	0.298
Ni Span C902	27×10^6	9.7×10^6	0.294

(Multiply lbf/in^2 by 6.89×10^{-6} to obtain GPa.)
(Multiply lbm/in^3 by 27.7×10^3 to obtain kg/m^3.)

[a]Properties vary with temperature. Compiled from various sources.
[b]PH—precipitation hardened.
Adapted from *Design Handbook*, 1987 Edition, Barnes Group (Associated Spring), Bristol, CT.

approximately 35% to 55%. The lower limit should be used for unconstrained designs (i.e., the spring diameter can be as large as necessary to keep the stresses low). The higher limit is used when space is limited and higher stresses are unavoidable.[5] The ultimate strength used to calculate the allowable stress can be found either through tabular information, such as can be found in Table 53.2, or can be calculated using Eq. 53.7. m is a constant based on the type of wire used for the spring: 0.163 for music wire, 0.193 for oil-tempered wire, 0.201 for hard-drawn wire, 0.155 for chrome vanadium wire, or 0.091 for chrome silicon wire. [Mechanical Springs]

Mechanical Springs

$$S_{ut} = \frac{A}{D_{\text{wire}}^m} \qquad 53.7$$

2. Selecting the allowable stress based on the yield strength in shear is probably the most theoretically rigorous method. The yield strength in shear can be calculated from the tensile yield strength using either the maximum shear stress or the distortion energy theory.[6] If called for, a factor of safety of approximately 1.5 is appropriate for ferrous springs.

3. Some specifications limit the torsional shear stress to a percentage of the tensile yield strength.

Some springs (e.g., flat leaf springs and helical torsion springs) experience a bending stress. Such springs are limited by the tensile yield strength of the spring material.

4. SAFE SPRING STRESSES: FATIGUE LOADING

Two methods can be used to design or analyze springs that are repeatedly stressed. The more rigorous method is to use a modified Goodman diagram. (The Wahl factor is applied to both the mean and alternating stress.) A simpler method is to design for static loading using a reduced maximum shear stress. An approximation of the endurance strength in shear can be calculated from the ultimate tensile strength (see Table 53.2) and a factor from Table 53.3. Since the table values are conservative, the lives of most springs designed to them will exceed the numbers of cycles listed. However, a factor of safety may still be used or required.

$$\tau_{\max} = \frac{S_e}{\text{FS}} = \frac{(\text{factor})S_{ut}}{\text{FS}} \qquad 53.8$$

[5]For highly precise springs with minimum hysteresis, creep, and drift, the percentages quoted in this section should be reduced.
[6]The tensile yield strength can be estimated for ferrous spring materials as 75% of the ultimate tensile strength.

Table 53.2 *Approximate Minimum Ultimate Tensile Strengths[a] of Typical Spring Materials (ksi, at room temperature[b])*

		wire size								
	in:	0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.30	0.40
material	mm:	0.5	1.0	1.5	2.0	2.5	3.8	5.1	7.6	10
high-carbon wire										
music wire ASTM A228		350	315	296	282	271	253	240		
hard-drawn ASTM A227		285	255	235	225	215	205	190	175	16
oil-tempered ASTM A229		295	265	250	235	230	215	195	180	17
valve spring ASTM A230						210	205	195		
alloy-steel wire										
chrome-vanadium SAE 6150,										
AISI 6150, ASTM A232			280	265	260	245	230	220	205	
stainless steel wire										
AISI 302, ASTM A313		300	275	260	245	235	215	190	160	
17-7 PH[c]		275	255	242	235	223	211			
18-2		296	270	265	255	253				
copper alloys										
phosphor bronze ASTM B159		150	140	138	135	132	130	128	122	

(Multiply in by 25.4 to obtain mm.)

(Multiply ksi by 6.895 to obtain MPa.)

[a]ASTM specifications provide a range of ultimate strengths. Table values approximately correspond to the minimum of the range.
[b]Properties vary with temperature.
[c]Precipitation hardened.
Adapted from *Design Handbook*, 1987 Edition, Barnes Group (Associated Spring), Bristol, CT.

Table 53.3 *Fatigue Correction Factor (fraction of ultimate tensile strength)*

	shear stress		bending stress	
fatigue life (cycles)	not shot-peened spring	shot-peened spring	not shot-peened spring	shot-peened spring
10,000	0.45*	0.45	0.80	0.80
100,000	0.35	0.42	0.53	0.62
1,000,000	0.33	0.40	0.50	0.60
10,000,000	0.30	0.36	0.48	0.58

*35% for phosphor bronze and type-302 stainless steel.
Adapted from *Design Handbook*, 1987 Edition, Barnes Group (Associated Spring), Bristol, CT.

5. HELICAL COMPRESSION SPRINGS: STATIC LOADING

The *spring index*, C, is the ratio of the mean coil diameter to the wire diameter.[7] (See Fig. 53.1.) It is difficult to wind springs with small (i.e., less than 4) spring indexes, and the operating stresses will be high. The wire cannot be easily bent to the desired small radius. On the other hand, springs with large indexes (i.e., greater than 12) are flimsy and tend to buckle. Most springs have indexes between 8 and 10, although 5 is a typical value for clutch springs.

Mechanical Springs

$$C = \frac{D_{\text{mean coil}}}{D_{\text{wire}}} \qquad 53.9$$

The mean coil diameter can be calculated as the average of the outside and inside diameters of the spring.

$$D_{\text{mean coil}} = \frac{D_i + D_o}{2} \qquad 53.10$$

The spring deflection can be written in terms of the mean coil diameter or the spring index.

$$x = \frac{F}{k} = \frac{8FD_{\text{mean coil}}^3 N}{GD_{\text{wire}}^4} = \frac{8FC^3N}{GD_{\text{wire}}} \qquad 53.11$$

[7]This section is only for helical compression springs manufactured from round wire. Springs can also be manufactured from wire with a square or rectangular cross section. The design equations are different in that case.